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COMPONENTS' IMPORTANCE MEASURES FOR INITIATING AND ENABLING EVENTS IN FAULT TREE ANALYSIS

New methods, applied to LBDD, for determining the basic events importance measures for unavailability and failure frequency analysis of coherent and non-coherent fault trees implemented in ASTRA 3

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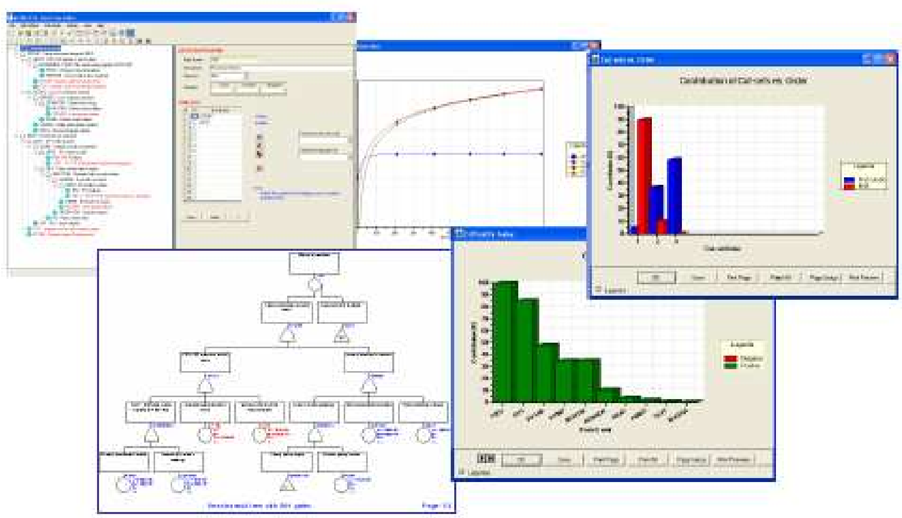

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SUMMARY

This report deals with the problem of determining the exact values of the importance indexes of basic events in case of both unavailability and frequency analysis of coherent and non-coherent fault trees. In particular a new method is described for determining the importance of enabling events in case of frequency analysis. Insights are given into the importance analysis implemented in the new software ASTRA 3.0 based on the Binary Decision Diagram approach with Labelled variables (LBDD). The analysis methods are also described with reference to modularised fault trees. Simple numerical examples are provided to clarify how the methods work. Proofs of the implemented equations are provided in Appendixes.

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1. INTRODUCTION

Fault Tree Analysis (FTA) is a popular methodology for Reliability, Availability, Maintainability, and Safety (RAMS) studies of complex systems, allowing to systematically describe the system's failure logic - for each system failure state or Top event - and to determine several probabilistic parameters useful e.g. for design improvement, diagnostic, maintenance. In particular the importance measures of basic events represent the contribution of components failure to the occurrence of the system failure described at the Top-event level.
Fault trees of real systems containing the AND, OR Boolean operators are referred to as *Coherent;* they are characterized by monotonic (non-decreasing) functions with respect to all basic events. Non monotonic logical functions are also of interest; the non monotonic behavior is due to the presence of the NOT operator. Fault trees containing the NOT operator are referred to as *non-coherent* and are very helpful in modelling e.g. mutually exclusive events, event-tree sequences, top-events conditioned to the working state of one or more component / subsystem, and maintenance procedures.

ASTRA allows the user to handle both coherent and non coherent fault trees. It is based on the state of the art approach of Binary Decision Diagrams (BDD) in which labels are dynamically associated with nodes giving what we called a Labelled BDD (LBDD). A clear description of the LBDD can be found in Contini & Matuzas (2010a), a report that provides also insight into the probabilistic analysis methods implemented in ASTRA 3.0.

This report is focussed on the methods implemented for the determination of the ranking of basic events according to a selected set of importance measures. The ranking of importance measures finds different applications, in particular for design improvement when coupled with sensitivity analysis techniques as described in Contini et al (2010c). Other applications are on system diagnosis and maintenance.
In literature several importance measures can be found; sometimes the same index is named in different ways by different scientists and practitioners. The importance measures considered in this report are:

- Marginal importance or probability of critical state;
- Criticality importance;
- Risk Achievement Worth;
- Risk Reduction Worth.

The selection of the importance measure of interest is up to the user and depends on the objectives of the analysis.

Whereas in the scientific literature the importance indexes based on unavailability are extensively described the importance based on the failure frequency (from which the Expected Number of Failures is determined and used as upper bound for unreliability) is not so rich. In case of failure frequency it is important to classify events as initiators or enablers since they role in systems are different and consequently they must be treated differently.

The aim of this report is to describe in detail the methods implemented in ASTRA 3.0 for calculating the importance ranking of basic events in case of both unavailability and frequency analysis. A new method has been developed and implemented for determining the exact importance value of enabling events and of the enabling contribution of initiating events.

The report is organised as follows. The next section describes some concepts useful to facilitate the reading of the report. Section 3 briefly describes the state of the art in importance analysis. Sections 4 and 5 constitute the core of the report and are devoted to the description of the analysis methods implemented in ASTRA 3.0. Proofs of equations and other useful information are given in four Appendixes.

2. BACKGROUND CONCEPTS

In order to facilitate the comprehension of the content of this report some basic concept are briefly described in this section. They concern: 1) the Labelled BDD implemented in ASTRA 3.0; 2) the equations for determining the unconditional failure and repair frequencies; and 3) the initiating and enabling events.

2.1 The binary decision diagram with labelled variables

A BDD with labelled variables (LBDD) is an Ordered BDD (OBDD) in which variables are dynamically labelled with the information about their type (Contini et al. 2006; Contini & Matuzas, 2010a). The LBDD was introduced to analyse non-coherent fault trees which generally contains three different types of events:

- Single form Positive (SP), i.e. events appearing in positive (normal) form only;
- Single form Negated (SN), i.e. events appearing in negated (complemented) form only;
- Double form (DP), i.e. events appearing in both forms (positive and negated).

The characterisation of three different types of variables requires labelling only two of them. Hence in ASTRA variables of SN type are labelled with the symbol $; variables of DF type are labelled with the symbol &. Additionally, the ordering used during the LBDD construction is &x<x<$x.
For instance the function $\phi = \bar{a}\, b + \bar{a}\, c + b\, \bar{c}$ contains the SN variable *a*, the SP variable *b* and the DF variable *c*. Hence the function is represented using the labelled variables as ϕ = $a b + $a c + b $c.
Events with the "&" label (DF type) are dynamically generated during the constructing of the LBDD when two occurrences of the same event - but differently labelled - are combined.
The rules for constructing the LBDD for non-coherent functions are described in Contini & Matuzas (2010a); for coherent functions the LBDD is obviously equivalent to the classical BDD.

The main reason for defining the LBDD is that different types of variables require different algorithms of analysis presenting different degree of complexity. Indeed, on nodes with &-variables the determination of the Prime Implicants (PI) and of the failure and repair frequencies require the logical intersection between the left and right descending functions, whereas this is not needed for the other two types of variables.
Moreover the knowledge of the variables' type is useful for the analysis of very large non-coherent fault trees, for which the complete LBDD is too complex to be stored in the available working memory. A set of rules, which have been defined in Contini & Matuzas (2010b), can be used for constructing a reduced ZBDD (RZBDD) of a non-coherent fault tree embedding only Significant Minimal Cut Sets (SMCS) having probability greater than and /or order less that predefined thresholds. A ZBDD is a convenient way to store MCS.

As an example of LBDD consider the function $\phi(\mathbf{x}) = x_2\,(x_1 + \bar{x}_3 + \bar{x}_4) + x_3\,(\bar{x}_1 + \bar{x}_2\, x_4)$. In this function all variables are of DF type. After labelling all negated variables the LBDD is constructed based on the ordering $x_2 < x_1 < x_3 < x_4$. Figure 2.1 shows the LBDD so obtained.
Note that in this LBDD there is one node with a DF variable ($\&x_2$) and one node with an SN variable ($\$x_1$), whereas the variables associated with all other nodes have no label, i.e. they behave as SP variables. Thus, in spite of the fact that the fault tree has all four variables of DF type, the LBDD has only one.
The above LBDD can be further simplified by applying the following rules:

- the & label can be removed if the left descendant is equal to 1 or the right one is 0;
- if the left descendant is equal to 0 or the right one is 1, the & label is substituted by $ and the two descendants are exchanged.

Changing $\&x_2$ with x_2, the final LBDD does not contain any variable of DF type.

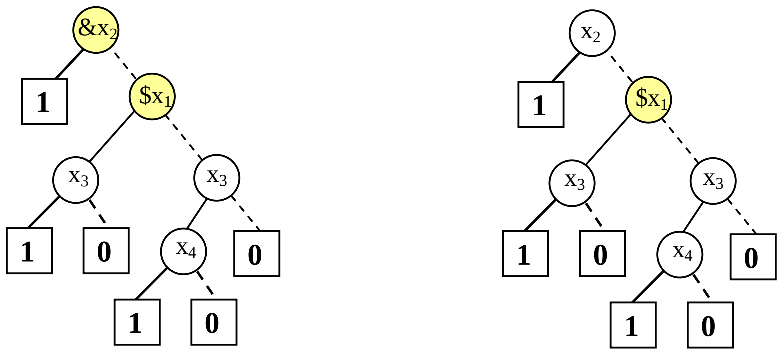


Figure 2.1 LBDD representation of Top = $x_2\ (x_1 + \overline{x}_3 + \overline{x}_4) + x_3\ (\overline{x}_1 + \overline{x}_2\ x_4)$.
Left: intermediate result: Right: final result

2.2 Determination of the unconditional failure and repair frequencies using LBDD

The importance measures described in this report are based on the unavailability and failure frequencies determined by visiting the LBDD in bottom-up mode.
The equations to be applied to each node depend on the type of the associated variable. The description of the equations to be applied on an LBDD is preceded by the definitions of failure and repair frequencies.

The time specific unconditional failure frequency $\omega_i(t)$ of a generic event x_i is the probability that x_i is verified ($x_i = 1$) at time t-t+dt given that $x_i = 0$ at time 0. If λ_i is the constant failure rate of the component, then $\omega_i(t) = \lambda_i\ (1 - q_i(t))$, where $q_i(t)$ is the component unavailability at time t. For non repairable components $\omega_i(t) = f(t)$, the failure density.

The time specific unconditional repair intensity $\nu_i(t)$ of a generic event x_i is the probability that x_i is not verified ($x_i = 0$) at time t-t+dt given that $x_i = 0$ at time 0. If μ_i is the constant repair rate of the component, then $\nu_i(t) = \mu_i\ q_i(t)$. For non repairable components $\mu_i = 0$, i.e. $\nu_i(t) = 0$.

Let (A B) be a system failure combination, i.e. a minimal cut set of a fault tree.
The unconditional failure frequency Ω(A B) that the combination (A B) occurs (enter into the failed state) in the time interval dt is given by the probability that A occurs in t-t+dt (represented by $\omega_A(t)$ dt) with B already failed at t (represented by $q_B(t)$) <u>or</u> that B occurs in t-t+dt with A already failed at t, i.e.:

$$\Omega(A\ B) = q_B(t)\ \ \omega_A(t) + q_A(t)\ \omega_B(t)$$

The unconditional repair frequency V(A B) in the time interval dt is given by the probability that A is repaired in t-t+dt (represented by $\nu_A(t)$) with B already failed at t (represented by $q_B(t)$) <u>or</u> that B is repaired in t-t+dt with A failed at t, i.e.:

$$V(a\ b) = q_B(t)\ \ \nu_A(t) + q_A(t)\ \nu_B(t)$$

With the aim of simplifying the notation from now on the dependence of time will be omitted, implicitly meaning that the equations are applied at a generic time t.

Let $X = x\ F + \overline{x}\ G$ be the generic node of the LBDD. The problem is to determine the unavailability and the unconditional failure and repair frequencies. The following notation is used:

- Q_{out} is the unavailability of the function X;

- ω_{out} is the unconditional failure frequency of X;
- ν_{out} is the unconditional repair frequency of X;
- $Q_1 = P(F)$ and $Q_0 = P(G)$;
- ω_1 and ω_0 are respectively the unconditional failure frequencies of F and G;
- ν_1 and ν_0 are respectively the unconditional repair frequencies of F and G;
- q_x, ω_x, and ν_x are respectively the unavailability, failure and repair frequencies of x.

At terminal nodes of the LBDD:
$Q_1 = 1$, $Q_0 = 0$, $\omega_1 = 0$, $\omega_0 = 0$, $\nu_1 = 0$, $\nu_0 = 0$.

The parameters Q_{out}, ω_{out} and ν_{out} at the root node determine the parameters at Top-event level.

The equations to be applied to nodes with variables of different type are listed herewith (Since the analysis is of bottom-up type the unavailability and frequencies of function F and G are supposed to be known).

<u>SP variable</u>
If x is coherent positive, then X = x F + G.
The unavailability is given by:

$$Q_{out} = q_x Q_1 + (1- q_x) Q_0 \tag{2.1}$$

Applying the rules for the determination of the failure frequency we get:

$$\omega_{out} = \omega_x Q_1 + \omega_1 q_x + \omega_0 (1- q_x) – \omega_x Q_0 \tag{2.2}$$

$$\nu_{out} = \nu_x Q_1 + \nu_1 q_x + \nu_0 (1- q_x) – \nu_x Q_0 \tag{2.3}$$

<u>SN variable</u>
If x is negated then X = $x F + G.
Hence the unavailability is given by:

$$Q_{out} = q_{\$x} Q_1 + (1- q_{\$x}) Q_0 \tag{2.4}$$

Applying the rules for the determination of the failure frequency, we get:

$$\omega_{out} = \nu_x Q_1 + (1 - q_x) \omega_1 + \omega_0 q_x – \nu_x Q_0 \tag{2.5}$$

$$\nu_{out} = \omega_x Q_1 + (1 - q_x) \nu_1 + \nu_0 (1- q_x) – \omega_x Q_0 \tag{2.6}$$

<u>DF variable</u>
If x is of DF type then X = x F + $x G + F G. In this case the product F G represents the consensus term and its importance for failure calculations.
The unavailability is given by:

$$Q_{out}(t) = q_x Q_1 + q_{\$x} Q_0 \tag{2.7}$$

Concerning the failure frequency:
Applying the rules for the determination of the failure frequency, we get:

$$\omega_{out} = \omega_x Q_1 + q_x \omega_1 + (1 - q_x) \omega_0 + \nu_x Q_0 – (\omega_x + \nu_x) \Pr \{F \wedge G \} \tag{2.8}$$

$$\nu_{out} = \nu_x Q_1 + q_x \nu_1 + (1 - q_x) \nu_0 + \omega_x Q_0 – (\omega_x + \nu_x) \Pr \{F \wedge G \} \tag{2.9}$$

NOTE:
In the above equations the dependence on time has not been displayed, meaning that the equations are applied at the generic time t, from t = 0 to the mission time t = T.

As can be seen equations (2.8-9) are more complex than equations (2.2-3) and (2.5-6). Using the LBDD the more complex equations are applied only when they are strictly necessary.

2.3 Initiating and enabling events

In fault tree analysis several types of events are considered. Along with the positive, negated and double form events there are situations in which it is necessary to distinguish between initiating and enabling events. This occurs when the Top-event describes a catastrophic system failure that is a failure that cannot be repaired or a failure with very dangerous consequences as e.g. “reactor explosion”, “release of toxic substance in the atmosphere”, “missile fails to perform its mission”. In these cases it is important to determine the probability of no failure during the mission time, i.e. the reliability or its complement to 1 (unreliability).

Note that the *exact* value of the unreliability of systems with repairable components cannot be determined by means of fault tree analysis (Clarotti, 1981). However (good) approximated conservative results can be obtained through the determination of the Expected Number of Failures (ENF). This bound is based on the unconditional failure frequency, which is the time derivative of the ENF (Kumamoto & Henley, 1996).

In performing the importance analysis based on the unconditional failure frequency it is necessary to subdivide the basic events into two groups: initiating and enabling because they have different meaning and they are treated differently. Initiating events cause perturbations of process variables; enabling events are associated with the failure on demand of the protective systems. For instance, in the following example: “An accident occurs if at the time of occurrence of the initiating event (failure mode of the control system causing a plant perturbation, e.g. very high pressure) the enabling event (failure mode of e.g. the shut-down system) has already occurred or it occurs at the time it is called to intervene”. The inverse sequence would lead to the plant shut down, but not to the accident.
This simple example shows that, differently from the unavailability analysis (failure at a given time t) where the order in which components fail is not relevant, in case of frequency analysis the failure sequence is very important.
The sequence of intervention of the initiator and enabler events can be modelled in ASTRA using the Inhibit (INH) gate, in which the two inputs can be complex dependent sub-trees.

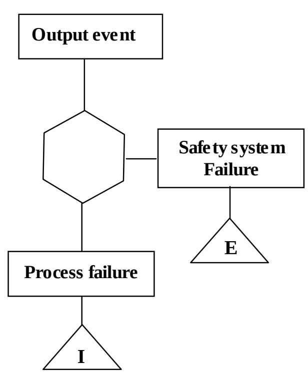


Figure 2.2 The INH gate as used in ASTRA for modelling the relationship between initiating and enabling events

An Initiating event is an event whose occurrence triggers the intervention of the Enabling event.
The output is true when, at the time the input is true, the condition defined by the enabler event is *already* true.
The method implemented in ASTRA identifies the events as either *initiators* or *enablers* depending on the sub-tree they belong to. Common events are flagged as initiators.

The differentiation of the type of events has an impact on the calculation of the ENF, since initiating events are characterized by their failure frequency $\omega(t)$, whereas enabling events, associated with components of the protective system, are characterized by their on-demand unavailability q(t). Hence the unconditional failure and repair frequencies of enabling events are set to zero.

Using the INH gate the failure and repair frequencies of the output event are given by:
$\omega_{out}(t) = \omega_I(t)\, q_E(t)$
$\nu_{out}(t) = \nu_I(t)\, q_E(t)$

In calculating the importance measures it is important to recognise that an initiating event may also appear, in certain MCS, as enabler. This situation arises when an initiating event is in a MCS with another initiating event.
Consider for instance the MCS (A B) where A, B are independent initiating events.

The failure frequency of the MCS is given by:
$\omega_{AB}(t) = \omega_A(t)\, q_B(t) + \omega_B(t)\, q_A(t)$

Consider e.g. event A. In the first term of the right hand side A behaves as initiator because it is characterised by $\omega_A(t)$, whereas in the second term as enabler, characterised by $q_A(t)$.

A description of the need to identify enabling events with a simple application can be found in Demichela et al (2003). It is worth to note that in that paper equation (3) is wrongly written: the typing mistake is pretty obvious. The correct equation is $\omega_c(t)\, dt = Q_c(t) \sum_{j=1}^{n} \frac{\omega_j(t)}{q_j(t)}\, dt$.

Other examples of application of the determination of the system failure considering initiating and enabling events are provided in the Test Case Report of ASTRA 3.0 (Contini & Matuzas, 2009).

3. STATE OF THE ART ON COMPONENTS' IMPORTANCE MEASURES

The importance measures of basic events in fault tree analysis allows the designer to identify the relatively most critical points of the system, for the top event of interest, from which design alternatives can be identified to improve the system performances. Other applications are system diagnoses and maintenance. A clear overview of importance measures can be found in Van der Borst & M., Schoonakker (2001),

For *coherent systems*, represented as a monotonic function of the vector of variables **x**, in the form $\Phi(\mathbf{x}) = x_k \ \Phi(x_k = 1, \mathbf{x}) + \Phi(x_k = 0, \mathbf{x})$, the first importance measure was proposed by Birnbaum. The Birnbaum importance [$IB_{xk}(t)$] of a component, say x_k, is defined as the probability that, at time t, the system is in a critical state for the failure of x_k, i.e. the system works if the component works and fails if the component fails.

Mathematically: $IB_{xk}(t) = P(\Phi(x_k = 1, \mathbf{x}, t)) - P(\Phi(x_k = 0, \mathbf{x}, t)) = \partial Q_\Phi(t) / \partial q_{xk}$ where $Q_\Phi(t)$ is the unavailability of $\Phi(\mathbf{x})$ at time t.

This index does not depend on the failure probability of component x. However, it is important that:

- It gives the maximum variation of the Top event unavailability when the component changes its state from perfectly working to failed;
- It is useful when used in connection with other indexes;
- Other indexes can be expressed as a function of it.

For non-coherent systems the Birnbaum index as defined above loses its meaning, since it can assume negative values. Non coherent systems are described by Boolean functions containing negated events.
The generalization of the Birnbaum index for *non-coherent* functions was proposed by Jackson, Zhang-Mei, Becker-Camarinopoulos and recently by Beeson-Andrews.

In order to be able to rank the events in order of importance Jackson (1983) proposed to use the absolute value of $IB_{xk}(t)$ for non-coherent systems, i.e.

$$IB_{xk}^{Jackson}(t) = \left| P(\Phi(x_k = 1, \mathbf{x}, t)) - P(\Phi(x_k = 0, \mathbf{x}, t)) \right| = \left| \partial Q_\Phi(t) / \partial q_{xk} \right|$$

Naturally, the absolute value implies a loss of information about the criticality of components.

Zhang and Mei (1985) defined the two probabilities $IB_{xk}^{+}(t)$ and $IB_{xk}^{-}(t)$ as representing the two contributions of the criticality of non-coherent variables:

$$IB_{xk}^{+}(t) = P[\Phi(x_k = 1, \mathbf{x}, t) - \Phi(x_k = 0, \mathbf{x}, t) = 1]$$

$$IB_{xk}^{-}(t) = P[\Phi(x_k = 1, \mathbf{x}, t) - \Phi(x_k = 0, \mathbf{x}, t) = -1]$$

Becker-Camarinopoulos (1993) introduced the definition of Failure Criticality Function (FCF) and Renewal Criticality Function (RCF).
The Failure Criticality Function for the generic variable x_k is a Boolean function defined as:
$FCF_{xk}(t) = \Phi(x_k = 1, \mathbf{x}, t)[1 - \Phi(x_k = 0, \mathbf{x}, t)]$. It expresses the fact that a generic component x_k is critical when the system fails if the component fails ($x_k = 1 \Rightarrow \Phi(x_k=1, \mathbf{x}) = 1$) AND it works if the component works ($x_k = 0 \Rightarrow \Phi(x_k=0, \mathbf{x}) = 0$, or equivalently $1 - \Phi(x_k=0, \mathbf{x}) = 1$).
If x_{xk} is coherent then $FCF_{xk} = IB_{xk}$, since $\Phi(x_k=1, \mathbf{x}) \ \Phi(x_k=0, \mathbf{x}) = \Phi(x_k=0, \mathbf{x})$.
The expected value of FCF_{xk} at time t, i.e. $Pr(FCF_{xk} = 1, t)$, is indicated as $p^f_{xk}(t)$ and represents the probability of the critical state for the failure of x_k at time t.
The Renewal Criticality Function RCF_{xk} is a Boolean function defined as:
$RCF_{xk}(t) = \Phi(x_k = 0, \mathbf{x}, t)[1 - \Phi(x_k = 1, \mathbf{x}, t)]$. It expresses the fact that a generic component x_k is critical when the system fails if the component is repaired ($x_k = 0 \Rightarrow 0 \Rightarrow \Phi(x_k = 0, \mathbf{x}) = 0$) AND it works if the component fails ($x_k = 1 \Rightarrow \Phi(x_k = 1, \mathbf{x}) = 0$, or equivalently $1 - \Phi(x_k = 1, \mathbf{x}) = 1$).

If x_{xk} is coherent then $RCF_{xk} = 0$ since $\Phi(x_k=1, \mathbf{x})\ \Phi(x_k=0, \mathbf{x}) = \Phi(x_k=0, \mathbf{x})$.
The expected value of RCF_{xk} at time t, $Pr(RCF_{xk} =1, t)$, is indicated as $p^r_{xk}(t)$ and represents the probability of the critical state for the repair of x_k at time t.

Beeson-Andrews (2003a) proposed an extension of the Birnbaum index of component importance for *non-coherent* systems. This measure is given by the sum of the probabilities of all critical states for the non-coherent component, i.e.:
$G_{xk}(t) = G_{xk}^F(t) + G_{xk}^R(t)$

– $G_{xk}^F(t)$ is the probability that, at time t, the system is in a working state such that the failure of component x in t-t+dt causes the system to fail. $G^F_{xk}(t) = \partial Q_\Phi(t) / \partial q_{xk}(t)$
– $G_{xk}^R(t)$ is the probability that the system is in a failed state at time t such that the repair of component x causes the system to fail. $G^F_{xk}(t) = \partial Q_\Phi(t) / \partial p_{xk}(t)$

In these equations $q_{xk}(t)=P(x_k=1)$ and $p_{xk}(t)=P(x_k=0)$.
The calculation of $G_{xk}^R(t)$ and $G_{xk}^F(t)$ is done considering the exact equation of the system unavailability calculated using the inclusion-exclusion method applied to the disjunction of the prime implicants or the BDD.

On the basis of the Birnbaum index, extended also to non-coherent functions, other indexes can easily be calculated such as the Criticality index, Risk Achievement Worth and Risk Reduction Worth as described in the next section.

Algorithms for determining importance measures working on the BDD representation of fault trees have been developed by Dutuit and Rauzy (2001).

The importance measures as defined above are all based on unavailability (failure probability at time t), i.e. the fault tree is analysed for a Top event concerning the system unavailability.
The scientific literature describing the determination of importance measures based on failure frequency is not as rich as for the case of unavailability. The first paper dates back to 1975 when Lambert (1975) introduced the definitions of initiating and enabling events. Initiating events cause perturbations of process variables; enabling events are associated with the on-demand unavailability of protective systems.

In IAEA TECDOC 590 (1991) the importance indexes of initiating and enabling events are defined as "*the ratio between the unconditional failure frequencies of the union of MCS that contain the event of interest over the Top event failure frequency".*
The use of the unconditional failure frequency instead of the Expected Number of Failure is simpler and faster compared with the use of the ENF in that the integration is not performed. These importance measures are almost equal to those calculated on the basis of the ENF only if the system failure frequency is constant, a condition that occurs when all (or almost all) system's components are repairable. Unfortunately, this is not the general case.

Concerning the importance measures of *initiating events* the first method was proposed by Barlow and Proschan (1975). Indeed the system failure frequency $\Omega_\Phi(t)$ can be expressed, for instance, using the Becker-Camarinopoulos notation, as:

$$\Omega_\Phi(t) = \sum_{i=1}^{n} \Omega_i(t) = \sum_{i}^{n} \ p_i^f(t)\ \omega_i(t)\ .$$

The importance of the i-th event is given by its contribution to system failure frequency, i.e.:

$$II_i(t) = \frac{\int_0^t \Omega_i(\tau)\, d\tau}{W_\Phi(0,t)}$$

This expression can easily be extended to non coherent functions, for which

$$\Omega_\Phi(t) = \sum_{i=1}^{n} \Omega_i(t) = \sum_{i=1}^{n} [\, p_i^f(t)\ \omega_i(t) + p_i^r(t)\, \nu_i(t)]$$

In the above equations *n* is the number of basic events and ω_i (ν_i) is the unconditional failure (repair) frequency of the i-th basic event.

The most recent method published is due to Beeson-Andrews (2003b). They described an *exact method* to determine the contribution to system failure of an enabling event when an initiating event causes the system to fail. Their method, applicable also to non-coherent function, is based on the determination of the second derivatives of the system unavailability with respect to the considered couple of initiating and the enabling events. This method is briefly described in this report by means of some examples.

A method alternative to that of Beeson-Andrews is proposed in this report. It allows determining the importance indexes of initiating and enabling events for both coherent and non-coherent functions. The importance measures are derived from the equation for determining the system's unconditional failure frequency. It is shown that the importance for initiating events is equal to the Barlow-Proschan index.
The complete description of the new method is given in section 5 together with some simple clarification examples.

The methods described in the next two sections have been implemented in ASTRA 3.0. In section 6 a comparison of the results of the analysis of a simple system performed by means of ASTRA 3.0 and of the previous version ASTRA 2.1 is provided. Differences are due to the fact that in ASTRA 2.1 the probabilistic analysis is performed on MCS and simplified equations substituted the integration of the failure frequency.

4. IMPORTANCE INDEXES BASED ON UNAVAILABILITY

In case of unavailability analysis, the following importance measures have been implemented in ASTRA 3.0 and described in this report:

- Marginal importance
- Criticality
- Risk Achievement Worth
- Risk Reduction Worth

The equations for determining the above importance measures will be derived considering the case of a non-coherent variable, from which the equations for coherent variables can easily be obtained.

4.1 Unavailability equation

Consider the following non-coherent function containing the variable x of DF type:

$$\Phi = x\,\Phi_1 + \bar{x}\,\Phi_0 \tag{4.1}$$

where $\Phi_1 = \Phi(x_i{=}1, \mathbf{x})$ and $\Phi_0 = \Phi(x_i{=}0, \mathbf{x})$.

In order to simplify the notation the time dependency of probabilities will not be explicitly shown, meaning that the equations given are supposed to be determined for a generic time t within the mission time.

The probability of function (4.1) is given by:

$$P(\Phi) = P(x)\,[P(\Phi_1) - P(\Phi_0)] + P(\Phi_0)$$

In this function, however, the contributions of x in normal form and in complemented form do not explicitly appear. In order to consider both forms it is convenient to add the consensus term $\Phi_1\,\Phi_0$ to equation (4.1) giving:

$$\Phi = x\,\Phi_1 + \bar{x}\,\Phi_0 + \Phi_1\,\Phi_0 \tag{4.2}$$

The unavailability of this function can be written in the following form in which the contributions to the unavailability of the event in positive and complemented form are made explicit:

$$P(\Phi) = P(x)\,[P(\Phi_1) - P(\Phi_1\,\Phi_0)] + P(\bar{x})\,[P(\Phi_0) - P(\Phi_1\,\Phi_0)] + P(\Phi_1\,\Phi_0)$$

Since $P(A\,\bar{B}) = P(A) - P(A\,B)$, the above can also be written as:

$$P(\Phi) = P(x)\,P(\Phi_1\,\overline{\Phi_0}) + P(\bar{x})\,P(\Phi_0\,\overline{\Phi_1}) + P(\Phi_1\,\Phi_0) \tag{4.3}$$

4.2 Marginal Importance Indexes

From eq. (4.3) we find the probability of the system critical state respectively for the failure and repair of event x:

$$\frac{\partial P(\Phi)}{\partial P(x)} = P(\Phi_1 \overline{\Phi_0}) \tag{4.4}$$

$$\frac{\partial P(\Phi)}{\partial P(\bar{x})} = P(\overline{\Phi_1} \Phi_0) \tag{4.5}$$

$p^f_x = P(\Phi_1 \overline{\Phi_0})$ is the probability of the system critical state for the failure of component x, i.e. the system works (the Top event is not verified) if the i-th component works and fails (the Top event is verified) if x fails.
For coherent variables $\Phi_0 \subset \Phi_1$ which leads to $\Phi_1 \Phi_0 = \Phi_0$ or equivalently $\overline{\Phi_1} \Phi_0 = 0$.

$p^r_x = P(\overline{\Phi_1} \Phi_0)$ is the probability of the system critical state for the repair of component x, i.e. the system works (the Top event is not verified) if the component x fails and fails (the Top event is verified) if x works.

Note that:

$p^f_{\bar{x}} = P(\Phi_{1\bar{x}} \overline{\Phi_{o\bar{x}}}) = P(\Phi_{0x} \overline{\Phi_{1x}}) = p^r_x$, simply because $\bar{x} = 1$ is equal to $x = 0$.

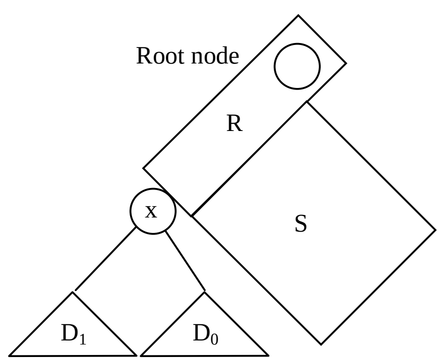
4.2.1 Implementation in ASTRA

For implementation purposes it was found more practical to use the equivalent form:

$P(\Phi_1 \overline{\Phi_0}) = P(\Phi_1) - P(\Phi_1 \Phi_0)$ and $\mathrm{P}(\overline{\Phi_1} \Phi_{01}) = \mathrm{P}(\Phi_0) - \mathrm{P}(\Phi_1 \Phi_0)$

These expressions are calculated in ASTRA for all events by traversing the LBDD twice.
Once p^f_i and p^r_i are known then all other indexes can be calculated.

The determination of the Marginal Importance indexes is briefly described below with reference to the following figure showing the different parts of interest of the BDD.



Consider the node with the variable x. D1 and D0 are the BDD descending respectively from x; R is the set of paths from x to the root node included; S is the set of all other paths (from 1 to root) not containing the node x. The above sets of nodes are not generally disjoint; only R has no common nodes with D_1, D_0.
Suppose that there is only one occurrence of x in the BDD.

We can write:

$\Phi = x\ R\ D_1 + \overline{x}\ R\ \Phi_0 + S$

$\Phi_1 = R\ D_1 + S$ and $\Phi_0 = R\ D_0 + S$.

$$P(\Phi_1\ \overline{\Phi_0}) = P(\Phi_1) - P(\Phi_1\ \Phi_0) = \mathrm{P(R)[P(D_1) - P(D_1\ D_0)]}$$

$$P(\overline{\Phi_1}\ \Phi_0) = P(\Phi_0) - P(\Phi_1\ \Phi_0) = \mathrm{P(R)[P(D_0) - P(D_1\ D_0)]}$$

The equations for determining p^f_i and p^r_i depend on the type of variable, i.e.:

1. if x is of type SP, then $\Phi_1\ \Phi_0 = \Phi_0$, which gives:

   $p^f_x = P(R)\,[P(D_1) - P(D_0)] \neq 0$

   $p^r_x = 0$

   Here p^f_i is nothing but the Birnbaum importance index of x.

2. if x is of type SN then, according to the classification of variables in ASTRA:

   $p^f_{\$x} = 0$

   $p^r_{\$x} = \mathrm{P(R)\,[P(D_1) - P(D_0)]}\quad ,0$

3. If x is of type DF then we have to consider the two contributors

   $p^f_x = P(R)\,[P(D_1) - P(D_1\ D_0)] \neq 0$

   $p^r_{\$x} = \mathrm{P(R)\,[P(D_1) - P(D_1\ D_0)]} \neq 0$

The determination of Pr(R), Pr(D_1), Pr{D_0} and Pr(D_1 D_0) for all nodes is obtained visiting the LBDD once upwards and once downwards.

4.2.2 Determination of Pr(D_1), Pr(D_0)

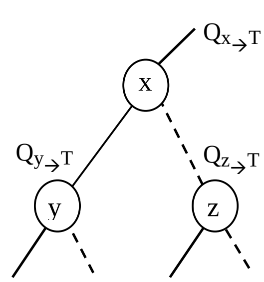
These values can easily be determined by visiting the LBDD upwards and applying, to each node the well known equation:

$Q = q_l\ Q_1 + q_r\ Q_0$

Values to be assigned to q_l and q_r depend on the type of variable. For SP and DF types
$q_l = q_x$ and $q_r = 1 - q_x$, whereas for SN variables $q_l = 1 - q_x$ and $q_r = q_x$.
For terminal nodes: $Q_1 = 1$ and $Q_0 = 0$.
At the root node the Top event unavailability Q_S is found

4.2.3 Determination of Pr(R)

The LBDD is visited once in top down mode to determine Pr(R) for all variables. Consider the generic node x in which $Q_{x \to T}$ represents the sum of the unavailability of all paths starting from the node x to the root (node x excluded).



The values to be associated to the descending nodes y and z are given by:

$$Q_{y \to T} = Q_{x \to T} / q_x$$

$$Q_{z \to T} = Q_{x \to T} /(1 - q_x)$$

The application of the above equations to all non terminal nodes allows associating the probability P(R) of the union of all paths from each node to the root, node excluded.

ASTRA gives, for DF variable the two contributions and their sum.

4.3 Structural importance I_{Sx}

The structural importance index is useful when probabilistic data for basic events are unavailable. Consequently, the only way to identify the relatively weak points of the system is to use the structural information. Some methods for determining the structural importance index can be found in literature. The one implemented in ASTRA 3.0 is due to Birnbaum. The structural indexes for the generic component x, I_{Sx} can be determined by means of the basic indexes $p^f{}_x$ and $p^r{}_x$ by setting probability 0.5 to all events.

4.4 Criticality index I_{Cx}

This index is defined as the relative variation of the Top event unavailability for a relative variation of the component failure / repair probability, i.e.:

$I_{Cx}(t) = [\partial Q_S(t) / \partial q_x(t)]\ [q_x(t) / Q_S(t)]$

<u>For SP variables,</u>
$I_{Cx}(t) = p^f{}_x(t)\ q_x(t) / Q_S(t)$ (4.6)

<u>For SN variables,</u>
$I_{C\$x}(t) = p^r{}_x(t)\ p_x(t) / Q_S(t)$ (4.7)
where $p_x(t) = 1 - q_x(t)$

<u>For DF variables</u>
The positive and the negative contributions are given by the above equations (4.6) and (4.7).

4.5 Risk Achievement Worth and Risk Reduction Worth

For coherent functions RAW_x is a measure of the risk increase when component x is assumed failed; it is defined as the ratio between the top event unavailability assuming event x failed $Q_S(t)|_{x=1}$ and $Q_S(t)$, i.e.

$RAW_x(t) = Q_S(t)|_{x=1} / Q_S(t)$

In calculating the RAW it is important to consider all other components that are dependent by the failure/removal of x.
The most important component (most critical) is the one with the highest RAW index.
From the definition it comes out that, for coherent variables: $1 < RAW_x(t) \leq 1/ Q_S(t)$

For coherent functions $RRW_x(t)$ is a measure of the risk reduction when the component x is assumed perfectly reliable:

$RRW_x(t) = Q_S(t) / Q_S(t)|_{x=0}$

The most important component is the one with the highest index value; when it works perfectly we have the maximum risk reduction. From the definition it follows that:
$1 < RRW_x(t) < \infty$

4.5.1 Relationships between RAW and RRW for coherent functions

RAW and RRW are related. In fact dividing by Qs(t) the equation

$Qs(t) = q_x(t)\, Qs(t)|_{x=1} + (1-q_x(t))\, Qs(t)|_{x=0}$, one gets:

$1 = q_x(t)\, RAW_x(t) + (1 - q_x(t)) / RRW_x(t)$

With simple algebraic manipulations:

$$RAW_x(t) = 1/ q_x(t)\, [1 - (1 - q_x(t)) / RRW_x(t)] \quad (4.8)$$

$$RRW_x(t) = (1- q_x(t))/ [1 - q_x(t)\, RAW_x(t)] \quad (4.9)$$

4.5.2 Relationships between RAW and RRW for non coherent functions

When dealing with non-coherent functions RAW and RRW must be calculated also for $\bar{x}$. It can be shown that the RAW of a negated variable is equal to the inverse of the RRW of the same variable in positive form:

$$RAW_{\bar{x}}(t) = Q_S(t)|_{\bar{x}=1} / Q_S(t) = Q_S(t)|_{x=0} / Q_S(t) = 1 / RRW_x(t) \quad (4.10)$$

Thus, $0 < RAW_{\bar{x}}(t) < 1$

Analogously, RRW of a negated variable is equal to the inverse of the RAW of the same variable in positive form:

$$RRW_{\bar{x}}(t) = Q_S(t) / Q_S(t)|_{\bar{x}=0} = Q_S(t) / Q_S(t)|_{x=1} = 1/ RAW_x(t) \quad (4.11)$$

Hence, $Q_S(t) < RRW_{\bar{x}}(t) < 1$

Therefore, for DF variables it is sufficient to determine the RAW and RRW for the positive form to obtain the same parameters for the negated form.

Moreover, dividing by Qs(t) the equation

$Qs(t) = q_{\bar{x}}(t)\, Qs(t)|_{\bar{x}=1} + (1-q_{\bar{x}}(t))\, Qs(t)|_{\bar{x}=0}$, one gets:

$1 = q_{\bar{x}}(t)\, RAW_{\bar{x}}(t) + (1 - q_{\bar{x}}(t)) / RRW_{\bar{x}}(t)$

With simple algebraic manipulations:

$$\mathrm{RRW}_{\bar{x}}(t) = [1/ q_x(t)]\, [1 / [1 – (1 - q_x(t))\, \mathrm{RAW}_{\bar{x}}(t)] \tag{4.12}$$

$$\mathrm{RAW}_{\bar{x}}(t) = [1/ (1 - q_x(t))]\, [1 – q_x(t) / \mathrm{RRW}_{\bar{x}}(t)] \tag{4.13}$$

4.5.3 Determination of RAW index for different types of variables

It is proved in Appendix 1 that both RAW and RRW, independently of the type of variable, can be based respectively on the probability of critical states for failure and repair $p^f_x(t)$ and $p^r_x(t)$:

Let x be a *variable* and $\Phi(\mathbf{x}) = x\, \Phi(1, \mathbf{x}) + \Phi(0, \mathbf{x})$ then:

$$\mathrm{RAW}_x(t) = 1 + (1- q_x(t))\, p^f_x(t) / Q_S(t) \tag{4.14}$$

Let x be an *SN variable* and $\phi(\mathbf{x}) = \$x\, \Phi(1, \mathbf{x}) + \Phi(0, \mathbf{x})$

$$\mathrm{RAW}_{\$x}(t) = 1 + (1- q_{\$x}(t))\, p^f_{\$x}(t) / Q_S(t) \tag{4.15}$$

Let x be a *DF variable* and $\phi(\mathbf{x}) = \&x\, \Phi(1, \mathbf{x}) + \overline{\&}x\, \Phi(0, \mathbf{x}) = x\, \Phi(1, \mathbf{x}) + \$x\, \Phi(0, \mathbf{x})$
In this case it is possible to determine both contributions, positive and negative keeping in mind that $\mathrm{RAW}_x + \mathrm{RRW}_{\$x}$ is not equal to $\mathrm{RAW}_{\&x}$, given by the above equations:

- Positive contribution: equation (4.14)
- Negative contribution: equation (4.15)

Note that $\mathrm{RAW}_x + \mathrm{RRW}_{\$x}$ is not equal to $\mathrm{RAW}_{\&x}$, as can easily be verified.

4.5.4 Determination of RRW index for different types of variables

Analogously to the RAW case the RRW equations are obtained.

If x be an *SP variable* of $\phi(\mathbf{x}) = x\, \phi(1, \mathbf{x}) + \phi(0, \mathbf{x})$ then:

$$\mathrm{RRW}_x(t) = 1 / [1 - q_x(t)\, p^f_x(t) / Q_S(t)] \tag{4.16}$$

If x is an *SN variable* then:

$$\mathrm{RRW}_{\$x}(t) = 1 / [1 – (1 - q_{\$x}(t))\, p^f_{\$x}(t) / Q_S(t)] \tag{4.17}$$

If &x is a *DF variable* and $\phi(\mathbf{x}) = \&x\, \Phi(1, \mathbf{x}) + \overline{\&}x\, \Phi(0, \mathbf{x}) = x\, \Phi(1, \mathbf{x}) + \$x\, \Phi(0, \mathbf{x})$ both contributions must be considered:

- Positive contribution: equation (4.16)
- Negative contribution: equation (4.17)

Note that $\mathrm{RAW}_x + \mathrm{RRW}_{\$x}$ is not equal to $\mathrm{RAW}_{\&x}$, as can easily be verified.

4.6 Determination of importance indexes on a modularised fault tree

An advantageous operation in fault tree analysis is the modularisation. The original function is decomposed into a set of simpler independent sub functions called modules. The remaining of the tree is the main module containing the Top event, also called Top module. The cost of the analysis, i.e. the computation time and the working memory requirements is generally lower than that of a non-

decomposed tree. The gain depends on the number and dimensions (number of variables) of the modules.
If the tree is modularised the algorithms of analysis can be independently applied to all modules and then the results can be recombined to obtain the final results at Top event level.

Given $\Phi = M\,\Phi_1 + \overline{M}\,\Phi_0 + \Phi_1\,\Phi_0$, where M is a module containing the variable x.

$$M = x\,M_1 + \overline{x}\,M_0 + M_1\,M_0$$

$$\overline{M} = x\,\overline{M_1} + \overline{x}\,\overline{M_0} + \overline{M_1}\,\overline{M_0}$$

$x \in \Phi$ is given by: $x \in M$ and $M \in \Phi$ or
$\overline{x} \in M$ and $\overline{M} \in \Phi$

$\overline{x} \in \Phi$ is given by: $x \in M$ and $\overline{M} \in \Phi$ or
$\overline{x} \in M$ and $M \in \Phi$

Let $p^f{}_M$ and $p^r{}_M$ be the probabilities of critical states for $M \in \Phi$.
Let $p^{fM}{}_x$ and $p^{rM}{}_x$ be the probabilities of critical states for $x \in M$.
Let $p^f{}_x$ and $p^r{}_x$ be the probabilities of critical states for $x \in \Phi$.

The indexes $p^f{}_x(t)$ and $p^r{}_x(t)$ for the generic variable x are obtained by combining $p^{fM}{}_x$ and $p^{rM}{}_x$ with the importance of the module M in the Top-module, represented as $p^f{}_M$ and $p^r{}_M$, by means of the following equations (see proof in Appendix2):

$$p^f{}_x = p^{fM}{}_x\, p^f{}_M + p^{rM}{}_x\, p^r{}_M \qquad (4.18)$$

$$p^r{}_x = p^{rM}{}_x\, p^f{}_M + p^{fM}{}_x\, p^r{}_M \qquad (4.19)$$

Equations 4.18 and 4.19 (see proof Appendix 2) take different forms depending on the type of variable x in M and M in Top as described in the following Table.

Table 4.1. Equations for determining $p^f{}_x$ and $p^r{}_x$ in a modularised LBDD

M∈Top / **x∈ M**	**SP** p^f_M	**SN** p^r_M	**DF** $p^f_M\,;\,p^r_M$
SP $p^f_x =$	$p^{fM}_x\ p^f_M$	---	$p^{fM}_x\ p^f_M$
$p^r_x =$	---	$p^{fM}_x\ p^r_M$	$p^{fM}_x\ p^r_M$
SN $p^f_x =$	---	$p^{rM}_x\ p^r_M$	$p^{rM}_x\ p^r_M$
$p^r_x =$	$p^{rM}_x\ p^f_M$	---	$p^{rM}_x\ p^f_M$
DF $p^f_x =$	$p^{fM}_x\ p^f_M$	$p^{rM}_x\ p^r_M$	$p^{fM}_x\ p^f_M + p^{rM}_x\ p^r_M$
$p^r_x =$	$p^{rM}_x\ p^f_M$	$p^{fM}_x\ p^r_M$	$p^{fM}_x\ p^r_M + p^{rM}_x\ p^f_M$

4.7 Example of application

The application of the above procedures is shown with reference to the simple non-coherent tree of Figure 1 containing a module. This modularised tree is described by means of two functions: the Top module and simple modules in both negated and normal forms.

$$Top = a\ b\ \overline{M} + a\ \overline{b}\ M$$

$$M = c\ \overline{d}$$

$$\overline{M} = \overline{c} + d$$

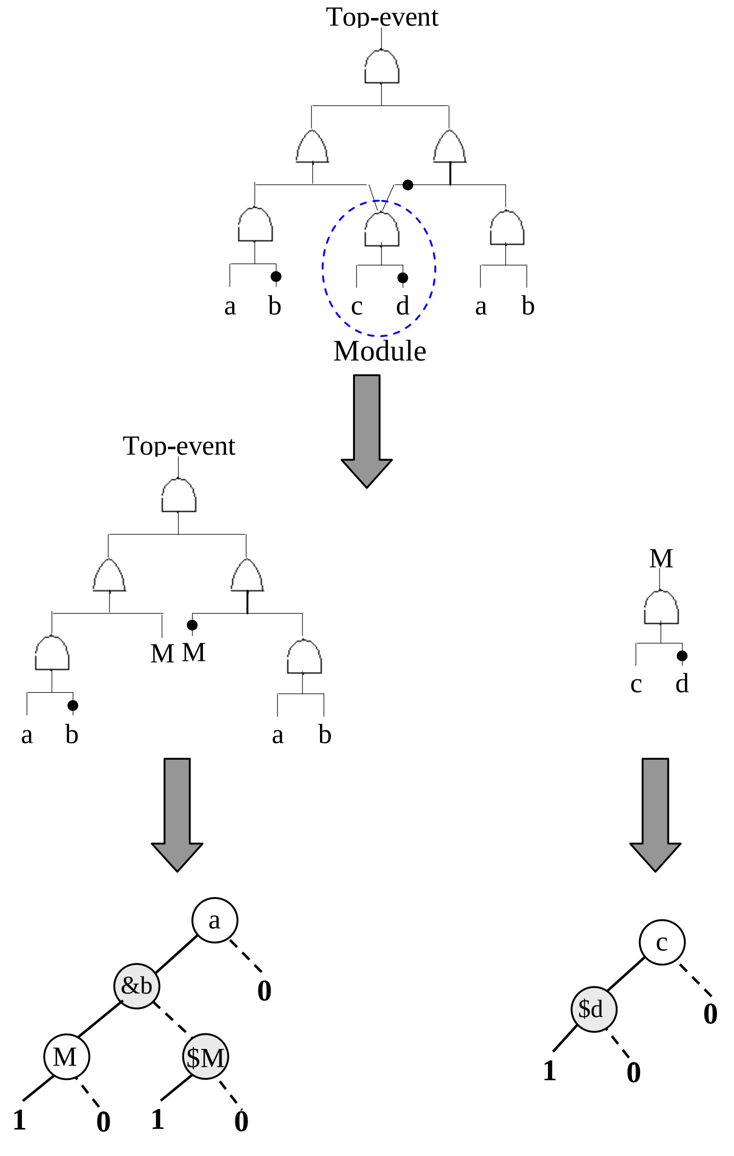


Figure 4.1 Non-coherent fault three, modularised tree and corresponding LBDD

This example is solved by means of the application of equations (4.18) and (4.19) on the LBDD. Then the same tree will be solved using the classical method based on MCS to show the correctness of the proposed equations.

For all variables in both functions the parameters $p^f(t)$ and $p^r(t)$ are determined. For the sake of simplicity the dependence on time is omitted. All events have the same failure probability. Since the example is simple the analytical solution is determined.

Analysis of M for determining q_M, $p^{f\,M}{}_x$, $p^{r\,M}{}_x$

$$M = c\,\bar{d} \qquad q_M = q_c\ (1 - q_d)$$
$$\bar{M} = \bar{c} + d \qquad q_{\$M} = 1 - q_c + q_c\, q_d$$

$c = 1 \Rightarrow M_{1c} = \bar{d}$ — $c = 0 \Rightarrow M_{oc} = 0$

$p^{f\,M}{}_c = P(M_{1c}) - P(M_{1c}\, M_{0c}) =1 - q_d$ — $p^{rM}{}_c = P(M_{0c}) - P(M_{1c}\, M_{0c}) = 0$

$d = 1 \Rightarrow M_{1d} = 0$ — $d = 0 \Rightarrow\ M_{0d} = c$

$p^{fM}{}_d = P(M_{1d}) - P(M_{1d}\, M_{0d}) = 0$ — $p^{rM}{}_d = P(M_{0d}) - P(M_{1d}\, M_{0d}) = q_c$

Analysis of the Top module for determining Q_S, $p^f{}_M$, $p^r{}_M$ as well as the importance of all other events not belonging to M.

$$Top = a\, b\ \bar{M} + a\ \bar{b}\, M$$
$$Q_{Top} = q_a\, q_b\, (1 - q_M) + q_a\, (1 - q_b)\ q_M$$

$a = 1 \Rightarrow Top_{1a} = b\,\bar{M} + \bar{b}\, M$ — $a = 0 \Rightarrow Top_{oa} = 0$

$p^f{}_a = q_b\, (1 - q_c + q_c\, q_d) + (1 - q_b)\, q_c\ (1 - q_d)$ — $p^r{}_a = 0$

$b = 1 \Rightarrow Top_{1b} = a\,\bar{M}$ — $b = 0 \Rightarrow Top_{ob} = a\, M$

$p^f{}_b = q_a\, (1 - q_c + q_c\, q_d)$ — $p^r{}_b = q_a\, q_c\, (1 - q_d)$

$M = 1 \Rightarrow Top_{1M} = a\,\bar{b}$ — $M = 0 \Rightarrow Top_{oM} = a\, b$

$p^f{}_M = q_a\, (1 - q_b)$ — $p^r{}_M = q_a\, q_b$

Application of equations (4.18) and (4.19) to events in the module M

.

$$p^f{}_c = p^{f\,M}{}_c\ p^f{}_M = (1 - q_d)\ q_a\ (1 - q_b)$$
$$p^r{}_c = p^{f\,M}{}_c\ p^r{}_M = (1 - q_d)\ q_a\ q_b$$
$$p^f{}_d = p^{r\,M}{}_d\ p^r{}_M = q_c\, q_a\, (1 - q_b)$$
$$p^r{}_d = p^{r\,M}{}_d\ p^f{}_M = q_c\, q_a\ q_b$$

The following table summarises the results

Event	$p^f{}_x$	$p^r{}_x$
a	$q_b\, (1 - q_c + q_c\, q_d) + (1 - q_b)\, q_c\ (1 - q_d)$	0
b	$q_a\, (1 - q_c + q_c\, q_d)$	$q_a\, q_c\ (1 - q_d)$
c	$(1 - q_d)\ q_a\ (1 - q_b)$	$(1 - q_d)\ q_a\ q_b$
d	$q_c\, q_a\, (1 - q_b)$	$q_c\, q_a\ q_b$

The correctness of the results in this table has been checked by hand based on the MCS.

5. IMPORTANCE INDEXES BASED ON FAILURE FREQUENCY

5.1 Determination of the unconditional failure frequency

Consider the following non-coherent function

$\Phi = x\,\Phi_1 + \overline{x}\,\Phi_0 + \Phi_1\,\Phi_0$

Since x is a variable of DF type then the consensus term is considered too, because it represents a valid implicant:

The failure frequency of Φ is given by:

$$\Omega(\Phi) = \Omega(x\,\Phi_1) + \Omega(\overline{x}\,\Phi_0) + \Omega(\Phi_1\,\Phi_0) - \Omega(x\,\Phi_1\,\Phi_0) - \Omega(\overline{x}\,\Phi_1\,\Phi_0)$$

Expanding the different terms:

$$\begin{aligned}\Omega(\Phi) = {} & \mathrm{P}(\mathrm{x})\,\Omega(\Phi_1) + \Omega(\mathrm{x})\,\mathrm{P}(\Phi_1) + \mathrm{P}(\overline{\mathrm{x}})\,\Omega(\Phi_0) + \Omega(\overline{\mathrm{x}})\,\mathrm{P}(\Phi_0) + \Omega(\Phi_1\,\Phi_0) + \\ & - \mathrm{P}(\mathrm{x})\,\Omega(\Phi_1\,\Phi_0) - \Omega(\mathrm{x})\,\mathrm{P}(\Phi_1\,\Phi_0) - \mathrm{P}(\overline{\mathrm{x}})\,\Omega(\Phi_1\,\Phi_0) - \Omega(\overline{\mathrm{x}})\,\mathrm{P}(\Phi_1\,\Phi_0)\end{aligned}$$

Rearranging:

$$\begin{aligned}\Omega(\Phi) = {} & \mathrm{P}(\mathrm{x})\,[\Omega(\Phi_1) - \Omega(\Phi_1\,\Phi_0)] + \Omega(\mathrm{x})\,[\mathrm{P}(\Phi_1) - \mathrm{P}(\Phi_1\,\Phi_0)] + \\ & + \mathrm{P}(\overline{\mathrm{x}})\,[\Omega(\Phi_0) - \Omega(\Phi_1\,\Phi_0)] + \Omega(\overline{\mathrm{x}})\,[\mathrm{P}(\Phi_0) - \mathrm{P}(\Phi_1\,\Phi_0)] + \Omega(\Phi_1\,\Phi_0)\end{aligned}$$

Since $\mathrm{P}(\mathrm{A}\,\overline{\mathrm{B}}) = \mathrm{P}(\mathrm{A}) - \mathrm{P}(\mathrm{A}\,\mathrm{B})$, the above equation can also be written as:

$$\begin{aligned}\Omega(\Phi) = {} & \mathrm{P}(\mathrm{x})\,\Omega(\Phi_1\,\overline{\Phi_0}) + \Omega(\mathrm{x})\,\mathrm{P}(\Phi_1\,\overline{\Phi_0}) + \mathrm{P}(\overline{\mathrm{x}})\,\Omega(\Phi_0\,\overline{\Phi_1}) \\ & + \Omega(\overline{\mathrm{x}})\,\mathrm{P}(\Phi_0\,\overline{\Phi_1}) + \Omega(\Phi_1\,\Phi_0)\end{aligned} \qquad (5.1)$$

From equations (5.1) it is straightforward to derive the equations for coherent functions. Indeed if x is coherent then $\Phi_1\,\Phi_0 = \Phi_0$, which also means that $\Phi_0\,\overline{\Phi_1} = 0$. Hence equation (5.1) becomes:

$$\Omega(\Phi) = \mathrm{P}(\mathrm{x})\,\Omega(\Phi_1\,\overline{\Phi_0}) + \Omega(\mathrm{x})\,\mathrm{P}(\Phi_1\,\overline{\Phi_0}) + \Omega(\Phi_0) \qquad (5.2)$$

5.2 Importance measures for initiating and enabling events

From equation (5.1) the following expressions are derived, from which the importance measures of initiating and enabling events are determined. They represent the probability/frequency of critical states. I and E indicate respectively Initiator and Enabler and the subscript represents the name (x) of the generic variable.

Equation (5.3) allows to determine the probability of the critical state for the occurrence of the initiating event x, i.e. the system fails in t-t+dt when the initiating event x occurs in t-t+dt. In other words the $I_x\,\omega_x$ represents the system failure frequency caused by the occurrence of x.

$$I_x = \frac{\partial\Omega(\Phi)}{\partial\Omega(x)} = P(\Phi_1\,\overline{\Phi_0}) \qquad (5.3)$$

Equation (5.4) represents the system failure frequency caused by an initiating event given that the enabler event x is failed. Hence E_x P(x) represents the contribution of the failure of x to the system failure frequency.

$$E_x = \frac{\partial \Omega(\Phi)}{\partial P(x)} = \Omega(\Phi_1\ \overline{\Phi_0}) \tag{5.4}$$

Equation (5.5) gives the probability of the critical state for the restoration of the event x, i.e. the system is failed in t-t+dt when the initiating event x is restored in t-t+dt. In other words $I_{\bar{x}}\ \nu_x$ represents the system failure frequency caused by the restoration of x.

$$I_{\bar{x}} = \frac{\partial \Omega(\Phi)}{\partial \Omega(\bar{x})} = P(\Phi_0\ \overline{\Phi_1}) \tag{5.5}$$

Finally equation (5.6) represents the system failure frequency caused by the repair of an initiating event given that the enabler event $\bar{x}$ has already occurred. Hence $E_{\bar{x}}\ q_{\bar{x}}$ is the contribution of the repair of to the system failure frequency:

$$E_{\bar{x}} = \frac{\partial \Omega(\Phi)}{\partial P(\bar{x})} = \Omega(\Phi_0\ \overline{\Phi_1}) \tag{5.6}$$

It has to be stressed that in applying the above equations, from (5.3) to (5.6), the negated variables resulting from $\Phi_1\ \overline{\Phi_0}$ and $\Phi_0\ \overline{\Phi_1}$ are characterised by their success probability only: their frequency must be set to zero. Indeed, *the negated part is not a real non coherence*, but more simply it represents a logical condition that must be satisfied for determining the logical function of the critical state. For this reason negated events cannot be considered as initiating events, which means that their failure frequency must set to zero.

Note that in equations 5.3 and 5.5 we find again the probabilities of critical states for failure and repair introduced in section 4, i.e. $I_x = p_x^f$ and $I_{\bar{x}} = p_x^r$.

The importance indexes, expressed with respect to the Expected Number of Failures (ENF), take the following form:

Initiator failure importance:

$$II_x = \int_0^t \omega_x(\tau)\ I_x(\tau)\, d\tau / W(0,t) \tag{5.7}$$

Enabler failure importance:

$$IE_x = \int_0^t q_x(\tau)\ E_x(\tau)\ d\tau / W(0,t) \tag{5.8}$$

where $W(0,t) = \int_0^t \Omega(\Phi,\tau)\ d\tau$ is the expected number of failures of the Top-event in which:

$$\Omega(\Phi,\tau) = \sum_{i=1}^{N} P(\Phi_1\ \overline{\Phi_0},\tau)\ \omega_i(\tau)$$

N is the number of basic events of the fault tree.

The index II_x represents the well known Barlow-Proschan importance index for initiating events.

If x is an initiating event, then IE_x is its contribution to the system failure frequency when another initiating event causes the system failure.
If x is an enabling event, then IE_x represents its importance index.

Analogously, for negated events:

Initiator repair importance:

$$II_{\bar{x}} = \int_0^t \nu_x(\tau)\, I_{\bar{x}}(\tau)\, d\tau / W(0,t) \tag{5.9}$$

Enabler repair importance:

$$IE_{\bar{x}} = \int_0^t (1 - q_x(\tau))\, E_{\bar{x}}(\tau)\, d\tau / W(0,t) \tag{5.10}$$

For non coherent fault trees the Expected Number of Failure $W(0,t) = \int_0^t \Omega(\Phi,\tau)\, d\tau$ considers also the repair as initiating events, i.e.:

$$\Omega(\Phi,\tau) = \sum_{i=1}^{Np} P(\Phi_1\ \overline{\Phi_0},\tau)\ \omega_i(\tau) + \sum_{i=1}^{Nn} P(\overline{\Phi_1}\ \Phi_0,\tau)\ \nu_i(\tau)$$

N_p and N_n are respectively the number of basic events in positive and negated form.

5.2.1 The Beeson-Andrews method for the determination of the importance of enabling events.

Beeson & Andrews proposed an exact method for determining the importance measures of initiating and enabling events based on the second partial derivative of the system unavailability. To our knowledge this is the first exact method published so far. For this reason this method will be considered as the reference method to show that our method gives the same results.

The main steps of the Beeson & Andrews are briefly described below for the coherent case and limited to the determination of the importance of enabling events or the importance of the enabling contribution of initiating events. As far as the importance of initiating events is concerned reference is made to the Barlow-Proschan method.

Given the Top event expressed as the disjunction of its MCS (or prime implicants) and the exact unavailability expression $Q_S(t)$ the first MCS containing x is considered.

$x_i \rightarrow x_j$ is a notation introduces here meaning that xi is the enabling event that contribute to the failure probability of Φ_S when the initiating event xj fails in t-t+dt leading to the system failure.

1. Determine the second partial derivative of Q_S with respect to xi and xj: $G_{i,j}(\tau) = \dfrac{\partial^2 Q_S(\tau)}{\partial q_{xi}(\tau)\, \partial q_{xj}(\tau)}$

2. Determine the unavailability Q_M, of the function M = Φ\ (MCS considered), i.e. the function without the considered MCS.

3. Determine the second partial derivative of Q_M with respect to xi and xj: $G_{M\,i,j}(\tau) = \dfrac{\partial^2 Q_{Mi,j}(\tau)}{\partial q_{xi}(\tau)\, \partial q_{xj}(\tau)}$

4. Calculate $E_{xi,xj}(\tau) = \dfrac{\partial^2 Q_S(\tau)}{\partial q_{xi}(\tau)\,\partial q_{xj}(\tau)}$ - $\dfrac{\partial^2 Q_{Mi,j}(\tau)}{\partial q_{xi}(\tau)\,\partial q_{xj}(\tau)}$

5. $IE_{xi,xj} = \dfrac{\int_0^t E_{xi,xj}(\tau)\, q_{xi}(\tau)\, \omega_{xj}(\tau)\, d\tau}{W_S(0,t)}$ ($W_S(0,t)$ is the ENF of the system for the mission time interval 0 **-** t)

Repeat steps 1-5 for all MCS containing xi and xj (i=1, 2,..).

The final result is given by: $IE_{xi} = \sum_j IE_{xi,xj}$

5.2.2 The ASTRA method for the determination of the importance of enabling events.

Let us recall the steps of our method for determining the importance of an enabling event or the enabling contribution to the importance of an initiating event.

<u>*Variable in positive form*</u>.

1. Determine $\Phi_{1x} = \Phi|_{x=1}$ and $\Phi_{0x} = \Phi|_{x=0}$

2. Determine $E_x = \Omega(\Phi_{1x}\ \overline{\Phi_{0x}})$ in such a way that negated variables in $\overline{\Phi_{0x}}$ have $v = 0$.

3. $IE_x = \dfrac{\int_0^t E_x\, q_x\, d\tau}{W_S(0,t)}$

<u>*Variable in negated form.*</u>

1. Determine $\Phi_{1x} = \Phi|_{x=1}$ and $\Phi_{0x} = \Phi|_{x=0}$

2. Determine $E_{\bar{x}} = \Omega(\Phi_{0x}\ \overline{\Phi_{1x}})$ in such a way that negated variables in $\overline{\Phi_{1x}}$ have $v = 0$.

3. $IE_{\bar{x}} = \dfrac{\int_0^t E_{\bar{x}}\, q_x\, d\tau}{W_\Phi(0,t)}$

Some examples of application of the above described methods are given below.

5.3 Some clarification examples

In this section some examples are provided to clarify the application of the ASTRA method for the determination of the importance indexes for initiating and enabler events, giving more emphasis to the latter because it is based on a new method.
The results obtained using the ASTRA method are compared with those determined by applying the exact Beeson-Andrews method.

5.3.1 First Example: coherent function

Consider the following coherent function:

$\Phi = (a + b)\ x + a\ c\ y$

Suppose that *x*, *y* are enabling events; *b*, *c* are initiating events; the event *a* has an enabling contribution to system failure since it is in combination with the initiating event *c* in the cut set (*a c y*). Analogous consideration holds for *c*.

Determination of IE_x, the importance of the enabling event x

<u>*Beeson-Andrews method.*</u>

Let us apply the above steps to the system function $\Phi = a\ x + b\ x + a\ c\ y$

$$Q_S = q_a\ q_x + q_b\ q_x + q_a\ q_c\ q_y - q_a\ q_b\ q_x - q_a\ q_c\ q_x\ q_y$$

Since x is an enabling event that belongs to MCS containing the initiating events a and b than x→ a and x→b are to be considered.

x→a

1: $\dfrac{\partial^2 Q_S}{\partial q_x \partial q_a} = 1 - q_b - q_c q_y$

2: $\Phi_M = b\ x + a\ c\ y \quad Q_M = q_b\ q_x + q_a\ q_c\ q_y - q_a\ q_b\ q_c\ q_x\ q_y$

3: $\dfrac{\partial^2 Q_M}{\partial q_x \partial q_a} = -\ q_b\ q_c\ q_y$

4: $\dfrac{\partial^2 Q_S}{\partial q_x \partial q_a} - \dfrac{\partial^2 Q_M}{\partial q_x \partial q_a} = 1 - q_b - q_c q_y + q_b\ q_c\ q_y$

5: $IE_{x,a} = \dfrac{\int_0^t [\omega_a\ [1 - q_b - q_c q_y + q_b\ q_c\ q_y]\ q_x\ d\tau}{W_S(0,t)}$

The dependence of probabilities and failure frequency on time is not shown for the sake of simplicity.

Now consider the second MCS containing x.

x→ b

1: $\dfrac{\partial^2 Q_S}{\partial q_x \partial q_b} = 1 - q_a$

2: $\Phi_M = a\ x + a\ c\ y \quad Q_M = q_a\ q_x + q_a\ q_c\ q_y - q_a\ q_c\ q_x\ q_y$

3: $\frac{\partial^2 Q_M}{\partial q_x \partial q_b} = 0$

4: $\frac{\partial^2 Q_S}{\partial q_x \partial q_b} - \frac{\partial^2 Q_M}{\partial q_x \partial q_b} = 1 - q_a$

5: $IE_{x,b} = \frac{\int_0^t [\omega_b \ (1-q_a)] \ q_x \ d\tau}{W_\Phi(0,t)}$

Since there is no other MCS containing x, the calculation ends: $IE_x = IE_{x,a} + IE_{x,b}$

ASTRA frequency-based method

This method is now applied to determine IE_x

$\Phi = (a + b) \ x + a \ c \ y$

1. $\Phi_{1x} = a + b$ and $\Phi_{0x} = a \ c \ y$

2. $\Phi_{1x} \ \overline{\Phi_{0x}} = a \ (\bar{c} + \bar{y}) + b \ (\bar{c} + \bar{y}) + \bar{a} \ b$

3. $\Omega(\Phi_{1x} \ \overline{\Phi_{0x}}) = \omega_a \ (1 - q_c \ q_y) + \omega_b \ (1 - q_a) - \omega_a \ q_b \ (1 - q_c \ q_y)$

4. $IE_x = \frac{\int_0^t [\omega_a \ (1-q_c \ q_y) + \omega_b \ (1-q_a) - \omega_a \ q_b \ (1-q_c \ q_y)] \ q_x(\tau) \ d\tau}{W_\Phi(0,t)}$

It is easy to verify the equivalence of this result with that previously calculated with the Beeson-Andrews method.

How the result of step 3 has been obtained is described in detail below.

$$\Omega(\Phi_{1x} \ \overline{\Phi_{0x}}) = \Omega[a \ (\bar{c} + \bar{y})] + \Omega[b \ (\bar{c} + \bar{y})] + \Omega[\bar{a} \ b] - \Omega[a \ b \ (\bar{c} + \bar{y})] - \Omega[\bar{a} \ b \ (\bar{c} + \bar{y})]$$

We elaborate the above expression without developing the terms containing negated variables.

$$\begin{aligned}\Omega(\Phi_{1x} \ \overline{\Phi_{0x}}) = {} & \omega_a \ P(\bar{c} + \bar{y}) + q_a \ \Omega(\bar{c} + \bar{y}) + \omega_b \ P(\bar{c} + \bar{y}) + q_b \ \Omega(\bar{c} + \bar{y}) + \omega_b \ P(\bar{a}) + q_b \ \Omega(\bar{a}) + \\ & - \omega_a \ q_b \ P(\bar{c} + \bar{y}) - \omega_b \ q_a \ P(\bar{c} + \bar{y}) - q_a \ q_b \ \Omega(\bar{c} + \bar{y}) - \omega_b \ P(\bar{a}) \ P(\bar{c} + \bar{y}) + \\ & - q_b \ \Omega(\bar{a}) \ P(\bar{c} + \bar{y}) - q_b \ P(\bar{a}) \ \Omega(\bar{c} + \bar{y}).\end{aligned}$$

Now we develop the probability of the negated terms, but not the frequency, and separate the two sub-expressions.

$$\Omega(\Phi_{1x}\ \overline{\Phi_{0x}}) = \omega_a\ (1-q_c\ q_y) + \omega_b\ (1-q_c\ q_y) + \omega_b\ (1-q_a) - \omega_a\ q_b\ (1-q_c\ q_y) - \omega_b\ q_a\ (1-q_c\ q_y) +$$
$$-\omega_b\ (1-q_a)\ (1-q_c\ q_y) +$$
$$+ q_a\ \Omega(\bar{c}+\bar{y}) + q_b\ \Omega(\bar{c}+\bar{y}) + q_b\ \Omega(\bar{a}) - q_a\ q_b\ \Omega(\bar{c}+\bar{y}) - q_b\ \Omega(\bar{a})\ (1-q_c\ q_y) +$$
$$-q_b\ (1-q_a)\ \Omega(\bar{c}+\bar{y}).$$

Now we "correct" this expression by setting Ω(logically negated variables) = 0, i.e. $\Omega(\bar{x}) = 0$, obtaining.

$$\Omega(\Phi_{1x}\ \overline{\Phi_{0x}}) = \omega_a\ (1-q_c\ q_y) + \omega_b\ (1-q_c\ q_y) + \omega_b\ (1-q_a) - \omega_a\ q_b\ (1-q_c\ q_y) +$$
$$-\omega_b\ q_a\ (1-q_c\ q_y) - \omega_b\ (1-q_a)\ (1-q_c\ q_y)$$

Rearranging and simplifying:

$$\Omega(\Phi_{1x}\ \overline{\Phi_{0x}}) = \omega_a\ (1-q_c\ q_y) + \omega_b\ (1-q_a) - \omega_a\ q_b\ (1-q_c\ q_y)$$

__Therefore the determination of__ $\Omega(\Phi_{1x}\ \overline{\Phi_{0x}})$ *__must be performed in such a way to consider, for negated events in__* $\bar{\Phi}_0$ *__only their probability.__*

Determination of the IE$_a$, i.e. the enabling contribution of the initiating event a

__Application of the Beeson-Andrews (BA) method.__

$$\Phi = a\ x + b\ x + a\ c\ y$$

$$Q_S = q_a\ q_x + q_b\ q_x + q_a\ q_c\ q_y - q_a\ q_b\ q_x - q_a\ q_c\ q_x\ q_y$$

a→c in MCS (a c y)

1: $\dfrac{\partial^2 Q_S}{\partial q_a \partial q_c} = q_y - q_x q_y$

2: $\Phi_M = (a+b)\ x$ $\quad\quad\quad Q_M = q_a\ q_x + q_b\ q_x - q_a\ q_b\ q_x$

3: $\dfrac{\partial^2 Q_M}{\partial q_a \partial q_c} = 0$

4: $\dfrac{\partial^2 Q_S}{\partial q_x \partial q_a} - \dfrac{\partial^2 Q_M}{\partial q_x \partial q_a} = q_y - q_x q_y$

5: $IE_{a,c} = \dfrac{\int_0^t (q_y - q_x q_y)\ q_a\ \omega_c\ d\tau}{W_\Phi(0,t)}$

Since *a* does not combine with any other initiating events in MCS the calculation terminates.

Application of the ASTRA frequency-based method

1: $\Phi_{1a} = \mathrm{x} + \mathrm{c\ y}$ and $\Phi_{0a} = \mathrm{b\ x}$

2: $\Phi_{1a}\ \overline{\Phi_{0a}} = \mathrm{x}\ \overline{\mathrm{b}} + \mathrm{c\ y}\ \overline{\mathrm{b}} + \mathrm{c\ y}\ \overline{\mathrm{x}}$

3: $\mathrm{E_a} = \Omega(\Phi_{1a}\ \overline{\Phi_{0a}}) = \omega_c\ q_y (1 - q_x)$

4: $$IE_a = \frac{\int_0^t \omega_c\ (q_y - q_x q_y)\ q_a\ d\tau}{W_\Phi(0,t)}$$

Also in this case we show the detail of the calculation performed at step 3.

$$\Omega(\Phi_{1a}\ \overline{\Phi_{0a}}) = \Omega(\mathrm{x}\ \overline{\mathrm{b}}) + \Omega(\mathrm{c\ y}\ \overline{\mathrm{b}}) + \Omega(\mathrm{c\ y}\ \overline{\mathrm{x}}) - \Omega(\mathrm{c\ x\ y}\ \overline{\mathrm{b}}) - \Omega(\mathrm{c\ y}\ \overline{\mathrm{b}}\ \overline{\mathrm{x}})$$

The determination of $\Omega(\Phi_{1a}\ \overline{\Phi_{0a}})$ *is performed setting to zero the failure frequency of negated events.* Note that, as previously mentioned, the frequency is set to zero also for all enabling events.
Note that $\Omega(\mathrm{x}\ \overline{\mathrm{b}})$ is set to zero because $\nu_b = 0$. But if x was combined with an initiating event in positive form the frequency would have been zero because $\nu_x = 0$.

$$\Omega(\Phi_{1a}\ \overline{\Phi_{0a}}) = \omega_c\ q_y(1-q_b) + \omega_c\ q_y(1-q_x) - \omega_c\ q_x\ q_y(1-q_b) - \omega_c\ q_y(1 - q_b - q_x + q_b\ q_x)$$

After simplifying, the final result is as follows: $\Omega(\Phi_{1a}\ \overline{\Phi_{0a}}) = \omega_c\ q_y(1-q_x)$

This expression, multiplied by q_a, gives the same results as the B-A method.

5.3.2 Second Example: coherent function

This example concerns the determination of the contribution to the system failure frequency considered by Beeson-Andrews (2003b).

$$\Phi = \mathrm{a\ b\ e + d\ e\ f + b\ g + d\ h}$$

Application of the Beeson-Andrews (BA) method.

The above authors determined, in their paper, the exact value of the enabling contribution of event d when the failure is caused by the event e, i.e. d→e. The result was:

$$I_{d,e} = \frac{\int_0^t [q_f\ (1-q_h)\ \Psi^*_{a,b,g}]\ q_d\ \omega_e\ d\tau}{W_{Top}(0,t)}$$

$$\Psi^*_{a,b,g} = 1 - q_a\ q_b - q_b\ q_g + q_a\ q_b\ q_g$$

Application of the ASTRA frequency-based method

We apply our method to determine the importance of d, supposing that all other events are initiating (no information is given in the BA paper about the types of events).

$$\Phi_{1d} = a\ b\ e + e\ f + b\ g + h$$

$$\Phi_{0d} = a\ b\ e + b\ g$$

$$\overline{\Phi_{0d}} = \overline{b} + \overline{a}\ \overline{g} + \overline{e}\ \overline{g}$$

$$\Phi_{1d}\ \overline{\Phi}_{0d} = e\ f\ (\overline{b} + \overline{a}\ \overline{g}) + h\ (\overline{b} + \overline{a}\ \overline{g}) + h\ \overline{e}\ \overline{g}$$

$$\begin{aligned}\Omega(\Phi_{1d}\overline{\Phi}_{0d}) = {} & \omega_e\ q_f P(\overline{b} + \overline{a}\ \overline{g}) + \omega_f\ q_e P(\overline{b} + \overline{a}\ \overline{g}) + \omega_h P(\overline{b} + \overline{a}\ \overline{g}) + \omega_h P(\overline{e}\ \overline{g}) + \\ & - \omega_e\ q_f q_h P(\overline{b} + \overline{a}\ \overline{g}) - \omega_f\ q_e q_h P(\overline{b} + \overline{a}\ \overline{g}) - \omega_h\ q_e q_f P(\overline{b} + \overline{a}\ \overline{g}) - \omega_h P(\overline{b}\ \overline{e}\ \overline{g} + \overline{a}\ \overline{g}\ \overline{e})\end{aligned}$$

This expression contains all contributions of d with all other initiating events, not only with e. If we consider only the terms with ω_e we get the contribution d→ e calculated by multiplying the corresponding terms by q_d:

$$I_{d,e} = \frac{\int_0^t [\ q_f P(\overline{b} + \overline{a}\ \overline{g})\ (1 - q_h)]\ q_d\ \omega_e\ d\tau}{W_{Top}(0,t)}$$

where $P\ (\overline{b} + \overline{a}\ \overline{g}) = 1 - q_a\ q_b - q_b\ q_g + q_a\ q_b\ q_g$ is equal to $\Psi^*_{a,b,g}$

The other contributions of q_d, not reported in the BA paper are as follows:

d→f

$$I_{d,f} = \frac{\int_0^t q_e\ \Psi^*_{a,b,g}\ (1 - q_h)\ q_d\ \omega_f\ d\tau}{W_{Top}(0,t)}$$

d→h

$$I_{d,h} = \frac{\int_0^t [\ \Psi^*_{a,b,g}\ (1 - q_e\ q_f) + \Psi_{a,b,e,g}\ ]\ q_d\ \omega_h\ d\tau}{W_{Top}(0,t)}$$

$$\Psi_{a,b,e,g} = q_a\ q_b (1 - q_e - q_g + q_e\ q_g)$$

This example clearly shows that the ASTRA frequency method is able to exactly calculate the importance index of enabling events in coherent functions.

5.3.3 Third example: a non coherent function

This example is taken again from Beeson Andrews (2003b).

$$\Phi = a\ b\ d + a\ \bar{b}\ c + \bar{c}\ d\ e + a\ d\ e + a\ c\ d$$

The example concerns the determination of the enabler importance of event b when c fails.

<u>*Application of the Beeson-Andrews method*</u>

b→c

$$IE_{b,c} = 0$$

$$IE_{\bar{b},c} = \frac{\int_0^t q_a\ (1\text{-}q_d)\ (1\text{-}q_b)\ \omega_c\ d\tau}{W_S(0,t)}$$

Being b a DF event, it has both a positive and a negative contribution. As can be seen from the results the positive contribution is 0 because the event b (in its positive form) does not appear in any prime implicant with c; the negative contribution is not zero because b (in its negated form) is contained in the second implicant (a $\bar{b}$ c) with c.

<u>*Application of the ASTRA frequency-based method.*</u>

Determination of the positive contribution, given by:

$$E_b = \Omega(\Phi_{1b}\overline{\Phi_{0b}})$$

$$\Phi_{1b} = a\ d + \bar{c}\ d\ e$$
$$\Phi_{0b} = a\ c + \bar{c}\ d\ e + a\ d\ e$$
$$\overline{\Phi_{0b}} = \bar{a}\ \bar{d} + \bar{a}\ \bar{e} + \bar{a}\ c + \bar{c}\ \bar{d} + \bar{c}\ \bar{e}$$
$$\Phi_{1b}\ \overline{\Phi_{0b}} = a\ d\ \bar{c}\ \bar{e}$$

$$E_b = \Omega(\Phi_{1b}\ \overline{\Phi_{0b}}) = \omega_a\ q_d\ P(\bar{c}\ \bar{e}) + \omega_d\ q_a\ P(\bar{c}\ \bar{e})$$

In this expression there are no terms containing ω_c; hence $E_{b,c} = 0$, that is $IE_{b,c} = 0$

Other contributions (not reported in the BA paper) are the following:

$$E_{b,a} = \omega_a\ q_d(1 - q_c - q_e + q_c\ q_e)$$
$$E_{b,d} = \omega_d\ q_a(1 - q_c - q_e + q_c\ q_e)$$

Determination of the negative contribution, given by:

$$E_{\bar{b}} = \Omega(\Phi_{1b}\overline{\Phi_{0b}})$$
$$\overline{\Phi_{1b}} = \bar{d} + \bar{a}\, c + \bar{a}\, \bar{e}$$
$$\Phi_{0b}\ \overline{\Phi_{1b}} = a\ c\ \bar{d}$$

$$E_{\bar{b}} = \Omega(\Phi_{0b}\ \overline{\Phi_{1b}}) = \omega_a\ q_c\ (1 - q_d) + \omega_c\ q_a\ (1 - q_d)$$

The term with ω_c allows obtaining the importance of b → c.

$$E_{\bar{b},c} = \Omega(\Phi_{1b}\overline{\Phi_{0b}}) = \omega_c\ q_a\ (1 - q_d)$$
$$IE_{\bar{b},c} = \frac{\int_0^t \omega_c\ q_a\ (1 - q_d)\ (1 - q_b)\ d\tau}{W_S(0,t)}$$

The result is the same as the one reported in the BA paper.

Besides this contribution there is also b → a:

$$E_{\bar{b},a} = \omega_a\ q_c\ (1 - q_d)$$
$$IE_{\bar{b},a} = \frac{\int_0^t \omega_a\ q_c\ (1 - q_d)\ (1 - q_b)\ d\tau}{W_S(0,t)}$$

From this example it can be seen that $\Phi_{1\bar{b}} = \Phi_{0b}$; $\Phi_{0\bar{b}} = \Phi_{1b}$; this property can be used to speed up the calculations.

5.3.4 Fourth example: another non-coherent function

This example considers the determination of the importance indexes of the two variables of an XOR function. These variables are both initiating but they also have the enabling contribution.

$$\Phi = a\ \bar{b} + \bar{a}\ b$$

<u>*Determination of the importance of initiating events*</u>

Determination of I_a

$$\Phi_{1a} = \bar{b} \text{ and } \Phi_{0a} = b \Rightarrow \Phi_{1a}\ \overline{\Phi_{0a}} = \bar{b}$$
$$I_a = P(\Phi_{1a}\ \overline{\Phi_{0a}}) = (1 - q_b)$$
$$I_a = \frac{\int_0^t (1 - q_b)\ \omega_a\ d\tau}{W_\Phi(0,t)}$$

Determination of $I_{\bar{a}}$

$$\Phi_{1a} = \bar{b} \text{ and } \Phi_{0a} = b \Rightarrow \Phi_{0a}\ \overline{\Phi_{1a}} = b$$

$I_{\bar{a}} = (\Phi_{0a}\ \overline{\Phi_{1a}}) = q_b$

$$I_{\bar{a}} = \frac{\int_0^t q_b\ \nu_a\ d\tau}{W_\Phi(0,t)}$$

Determination of I_b

$\Phi_{1b} = \bar{a}$ and $\Phi_{0b} = a \Rightarrow \Phi_{1b}\ \overline{\Phi_{0b}} = \bar{a}$

$I_b = P(\Phi_{1b}\ \overline{\Phi_{0b}}) = 1 - q_a$

$$I_b = \frac{\int_0^t (1-q_a)\ \omega_b\ d\tau}{W_\Phi(0,t)}$$

Determination of $I_{\bar{b}}$

$\Phi_{1b} = \bar{a}$ and $\Phi_{0b} = a \Rightarrow \Phi_{0b}\ \overline{\Phi_{1b}} = a$

$I_{\bar{b}} = P(\Phi_{0b}\ \overline{\Phi_{1b}}) = q_a$

$$I_{\bar{b}} = \frac{\int_0^t q_a\ \nu_b\ d\tau}{W_\Phi(0,t)}$$

<u>Determination of the enabling contribution of initiating events</u>

Determination of IE_a

$\Phi_{1a} = \bar{b}$ and $\Phi_{0a} = b \Rightarrow \Phi_{1a}\ \overline{\Phi_{0a}} = \bar{b}$

$E_a = \Omega(\Phi_{1a}\ \overline{\Phi_{0a}}) = \nu_b$

$$IE_a = \frac{\int_0^t q_a\ \nu_b\ d\tau}{W_\Phi(0,t)}$$

Determination of $IE_{\bar{a}}$

$\Phi_{1a} = \bar{b}$ and $\Phi_{0a} = b \Rightarrow \Phi_{0a}\ \overline{\Phi_{1a}} = b$

$E_{\bar{a}} = (\Phi_{0a}\ \overline{\Phi_{1a}}) = \omega_b$

$$IE_{\bar{a}} = \frac{\int_0^t (1-q_a)\ \omega_b\ d\tau}{W_\Phi(0,t)}$$

Determination of IE_b

$\Phi_{1b} = \bar{a}$ and $\Phi_{0b} = a \Rightarrow \Phi_{1b}\ \overline{\Phi_{0b}} = \bar{a}$

$E_b\Omega = (\Phi_{1b}\ \overline{\Phi_{0b}}) = \nu_a$

$$IE_b = \frac{\int_0^t q_b \, \nu_a \, d\tau}{W_\Phi(0,t)}$$

Determination of $IE_{\bar{b}}$

$\Phi_{1b} = \bar{a}$ and $\Phi_{0b} = a \Rightarrow \Phi_{0b} \, \overline{\Phi_{1b}} = a$

$E_{\bar{b}} = \Omega(\Phi_{0b} \, \overline{\Phi_{1b}}) = \omega_a$

$$IE_{\bar{b}} = \frac{\int_0^t (1 - q_b) \, \omega_a \, d\tau}{W_\Phi(0,t)}$$

From this example it can be seen that:

$I_a = IE_{\bar{b}}$ and $I_{\bar{a}} = IE_b$ and vice versa exchanging a with b. This property can be used to speed up the calculations.

5.4 Determination of the importance indexes of initiating and enabling events on a modularised fault tree

If the tree is modularised the algorithms of analysis can be independently applied to all modules and then the results can be recombined to obtain the final results at Top event level.
In this section the equations are given with reference to the more general case of a non-coherent function. Proofs are provided in Appendix 3.

Given $\Phi = M\,\Phi_1 + \overline{M}\,\Phi_0 + \Phi_1\,\Phi_0$, where M is a module containing the variable x.

$$M = x\,M_1 + \bar{x}\,M_0 + M_1\,M_0$$

$$\overline{M} = x\,\overline{M_1} + \bar{x}\,\overline{M_0} + \overline{M_1}\,\overline{M_0}$$

$x \in \Phi$ is given by: $x \in M$ and $M \in \Phi$ or
$\bar{x} \in M$ and $\overline{M} \in \Phi$

$\bar{x} \in \Phi$ is given by: $x \in M$ and $\overline{M} \in \Phi$ or
$\bar{x} \in M$ and $M \in \Phi$

The above conditions lead to the relationships for determining the integrand function of importance indexes of initiating events as described in Table 5.1 and the relationships for determining the integrand functions of the importance of enabling events as in Table 5.2.
The importance indexes are finally obtained by applying equations from (5.7) to (5.10).

Table 5.1. Equations for determining I_x and $I_{\bar{x}}$ in a modularised LBDD

M∈ Top / **x∈ M**	**SP** I_M	**SN** $I_{\overline{M}}$	**DF** $I_M \, ; I_{\overline{M}}$
SP			
$I_x =$	$I_x^M\ I_M$	---	$I_x^M\ I_M$
$I_{\bar{x}} =$	---	$I_x^M\ I_{\overline{M}}$	$I_x^M\ I_{\overline{M}}$
SN			
$I_x =$	---	$I_{\bar{x}}^M\ I_{\overline{M}}$	$I_x^M\ I_{\overline{M}}$
$I_{\bar{x}} =$	$I_{\bar{x}}^M\ I_M$	---	$I_{\bar{x}}^M\ I_M$
DF			
$I_x =$	$I_x^M\ I_M$	$I_{\bar{x}}^M\ I_{\overline{M}}$	$I_x^M\ I_M + I_{\bar{x}}^M\ I_{\overline{M}}$
$I_{\bar{x}} =$	$I_{\bar{x}}^M\ I_M$	$I_x^M\ I_{\overline{M}}$	$I_{\bar{x}}^M\ I_M + I_x^M\ I_{\overline{M}}$

Table 5.2. Equations for determining E_x and $E_{\bar{x}}$ in a modularised LBDD

x∈ M \ **M∈Top**	**SP** E_M	**SN** $E_{\bar{M}}$	**DF** $E_M ; E_{\bar{M}}$
SP $E_x =$ $E_{\bar{x}} =$	$I_x^M \; E_M$ ---	--- $I_x^M \; E_{\bar{M}}$	$I_x^M \; E_M$ $I_x^M \; E_{\bar{M}}$
SN $E_x =$ $E_{\bar{x}} =$	--- $I_{\bar{x}}^M \; E_M$	$I_{\bar{x}}^M \; E_{\bar{M}}$ ---	$I_{\bar{x}}^M \; E_{\bar{M}}$ $I_{\bar{x}}^M \; E_M$
DF $E_x =$ $E_{\bar{x}} =$	$I_x^M \; E_M$ $I_{\bar{x}}^M \; E_M$	$I_{\bar{x}}^M \; E_{\bar{M}}$ $I_x^M \; E_{\bar{M}}$	$I_x^M \; E_M + I_{\bar{x}}^M \; E_{\bar{M}}$ $I_{\bar{x}}^M \; E_M + I_x^M \; E_{\bar{M}}$

5.4.1 Example of application

The application of the above procedures is shown with reference to the simple non-coherent tree of Figure 5.1 containing a module.

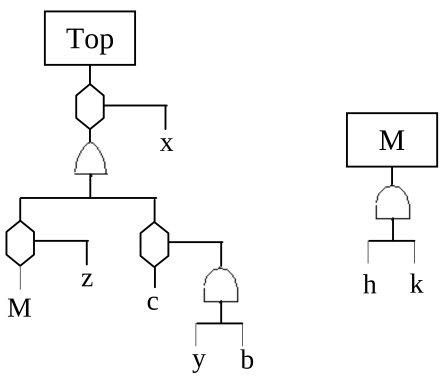


Figure 5.1. Sample Fault tree with a simple module

In ASTRA the above fault tree is transformed into the one in Figure 5.2, in which the gates INH are replaced by AND gates; all events of the protection system are identified and labelled as enabler. The resulting function is: $\Phi = x\,(z\,M + c\,y\,b)$

The types of variables are identified; they are:

- initiating and coherent: c, h, k
- enabling and coherent: b, x, y, and z;

Before determining the importance measures of events in M it is necessary to determine the unavailability and expected number of failure of M.

$Q_M = q_h \; q_k$

$$W_M = \int_0^t (\omega_h\ q_k + \omega_k\ q_h)\ d\tau$$

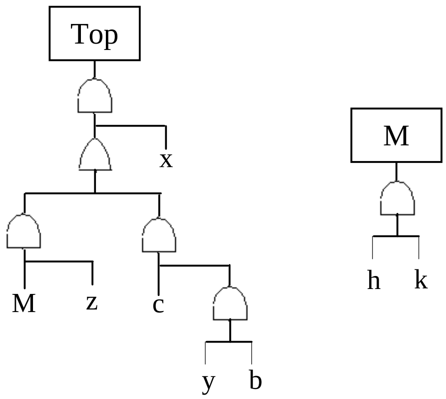


Figure 5.2. Fault tree of Figure 5.1 after removing INH gates

Analysis of the events h and k in M

Since h and k are both initiating events, their importance measures are calculated through the marginal importance p_h^{fM}, p_k^{fM} which can be determined as described in section 4.

$$II_h^M = \frac{\int_0^t p_h^{fM}(\tau)\ \omega_h(\tau)\ d\tau}{W_M(0,t)} \qquad II_k^M = \frac{\int_0^t p_k^{fM}(\tau)\ \omega_k(\tau)\ d\tau}{W_M(0,t)}$$

Analogously the importance of the module M in the Top module is given by:

$$II_M = \frac{\int_0^t p_M^f(\tau)\ \omega_M(\tau)\ d\tau}{W_\Phi(0,t)}$$

The importance index is given by composing the above indexes, i.e.:
$\mathrm{II_h = II_h^M\ II_M}$ and $\mathrm{II_k = II_k^M\ II_M}$

The contributions of these events to system failure frequency when another initiating event causes the system failure are calculated using the ASTRA method.

$$\mathrm{E_h = \Omega(M_{1h}\ \overline{M_{0h}})}$$
$$\mathrm{E_k = \Omega(M_{1k}\ \overline{M_{0k}})}$$
$$\mathrm{E_M = \Omega(\Phi_{1M}\ \overline{\Phi_{0M}})}$$

Results are as follows:

$$IE_h = \frac{\int_0^t \omega_k\ q_h(\tau)\ d\tau}{W_M(0,t)}$$

$$IE_k = \frac{\int_0^t \omega_h \; q_k(\tau)\; d\tau}{W_M(0,t)}$$

$IE_M = 0$ Indeed $\Omega(\Phi_{1M}\;\overline{\Phi_{0M}}) = \Omega(x\; z(\bar{c} + \bar{y} + \bar{b})) = 0$ since x and z are enablers ($\omega = 0$).

Therefore:
$E_h = E_h^M \; I_M = \omega_k \; q_h \; p_h^{fM} \; p_M^f$
$E_k = E_k^M \; I_M = \omega_h \; q_k \; p_k^{fM} \; p_M^f$

Analysis of c
The initiating event c in the Top-module is coherent. Then:

$$II_c = \frac{\int_0^t p_c^f(\tau)\; \omega_c(\tau)\; d\tau}{W_\Phi(0,t)}$$

$E_c = \Omega(\Phi_{1c}\;\overline{\Phi_{0c}}) = 0$

Analysis of b
This event is enabler:

$\Phi_{1b} = x(z\; M + c\; y)$

$\Phi_{0b} = x\; z\; M \Rightarrow \overline{\Phi_{0x}} = \bar{x} + \bar{z} + \overline{M}$

$\Phi_{1b}\overline{\Phi_{ob}} = x\; c\; y\; (\bar{z} + \overline{M})$

$E_b = \Omega(\Phi_{1b}\;\overline{\Phi_{0b}}) = \omega_c\; q_y\; q_x\; (1 - q_z)\; (1 - q_M)$

$$IE_b = \frac{\int_0^t [\omega_c\; q_y\; q_x\; (1 - q_z)\; (1 - q_M)]\; q_b(\tau)\; d\tau}{W_\Phi(0,t)}$$

Analysis of x

$\Phi_{1x} = z\; M + c\; y\; b$

$\Phi_{0x} = 0 \Rightarrow \overline{\Phi_{0x}} = 1$

Therefore: $\Omega(\Phi_{1x}\overline{\Phi}_{0x}) = z\; M + c\; y\; b = \omega_M\; q_z(1 - q_b q_c q_y) + (\omega_c\; q_b q_y + \omega_b\; q_c q_y)\; (1 - q_M q_z)$

$$IE_x = \frac{\int_0^t [\omega_M \ q_z (1 - q_b q_c q_y) + (\omega_c \ q_b q_y + \omega_b \ q_c q_y)\ (1 - q_M q_z)]\ q_x(\tau)\ d\tau}{W_\Phi(0,t)}$$

Analysis of y

$\Phi_{1y} = \mathrm{x(z\ M + c\ b)}$

$\Phi_{0y} = \mathrm{x\ z\ M} \Rightarrow \overline{\Phi_{0y}} = \overline{\mathrm{x}} + \overline{\mathrm{z}} + \overline{\mathrm{M}}$

$\Phi_{1y}\overline{\Phi_{oy}} = \mathrm{x\ c\ b\ (\overline{z} + \overline{M})}$

Therefore: $\Omega(\Phi_{1y}\overline{\Phi}_{0y}) = (\omega_c\ q_b\ q_x + \omega_b\ q_c\ q_x)\ (1 - q_z - q_M + q_z q_M)$

$$IE_y = \frac{\int_0^t (\omega_c \ q_b \ q_x + \omega_b \ q_c \ q_x)\ (1 - q_z - q_M + q_z q_M)\ q_y(\tau)\ d\tau}{W_\Phi(0,t)}$$

Analysis of z

$\Phi_{1z} = \mathrm{x(M + c\ y\ b)}$

$\Phi_{0z} = \mathrm{x\ c\ y\ b} \Rightarrow \overline{\Phi_{0z}} = \overline{\mathrm{x}} + \overline{\mathrm{c}} + \overline{\mathrm{y}} + \overline{\mathrm{b}}$

$\Phi_{1y}\overline{\Phi_{oy}} = \mathrm{x\ M\ (\overline{c} + \overline{y} + \overline{b})}$

Therefore: $\Omega(\Phi_{1z}\overline{\Phi}_{0z}) = \omega_M\ q_x[(1 - q_c) + (1 - q_y) + (1 - q_b)]$

$$IE_y^z = \frac{\int_0^t \omega_M \ q_x[(1 - q_c) + (1 - q_y) + (1 - q_b)]\ q_z(\tau)\ d\tau}{W_\Phi(0,t)}$$

The following table summarises the results

Event	**Type**	**II**	**IE**
b	enabling	---	$IE_b > 0$
c	initiating	$II_c > 0$	$IE_c = 0$
h	“	$II_h = II_h^M\ II_M$	$E_h = E_h^M\ I_M$
k	“	$II_k = II_k^M\ II_M$	$E_k = E_k^M\ I_M$
x	enabling	---	$IE_x > 0$
y	“	---	$IE_y > 0$
z	“	---	$IE_z > 0$

5.5 Risk Achievement Worth (RAW) and Risk Reduction Worth (RRW)

RAW and RRW can easily be determined after determining $\Omega(\Phi_1)$ and $\Omega(\Phi_0)$

$$RAW_x(t) = \frac{\int_0^t \Omega(\Phi_1, \tau)\, d\tau}{W_\Phi(0,t)} \tag{5.11}$$

$$RRW_x(t) = \frac{W_\Phi(0,t)}{\int_0^t \Omega(\Phi_0, \tau)\, d\tau} \tag{5.12}$$

The determination of $\Omega(\Phi_1, \tau)$ and $\Omega(\Phi_0, \tau)$ can be performed by visiting the BDD in bottom up way for <u>each</u> basic event and for each time τ.

5.6 Implementation issues

5.6.1 Importance of initiating events.

As mentioned in 5.1 the importance of initiating events, respectively in positive and negated form is given by:

$$II_x = \int_0^t \frac{\omega_x(\tau)\, I_x(\tau)}{\Omega(\Phi, \tau)} d\tau \tag{5.13}$$

$$II_{\bar{x}} = \int_0^t \frac{\nu_x(\tau)\, I_{\bar{x}}(\tau)}{\Omega(\Phi, \tau)}\, d\tau \tag{5.14}$$

In these equations $I_x(\tau)$ and $I_{\bar{x}}(\tau)$ are nothing but the probabilities of critical states for failure and repair of x, represented in section 4 as p_x^f and p_x^r. Since these parameters have already been implemented in ASTRA the problem of determining the importance indexes of initiating events is straightforward.

5.6.2 Importance of enabling events

Let x be an enabling event. The determination of the Criticality importance index for enabling events requires the calculation of $E_x = \Omega(\Phi_1\, \overline{\Phi_0})$ and / or $E_{\bar{x}} = \Omega(\Phi_0\, \overline{\Phi_1})$

The determination of the exact values of the importance of enabling events, e.g. for $E_x = \Omega(\Phi_1\, \overline{\Phi_0})$, can be done following the definition:

1. Determine Φ_1;
2. Determine Φ_0;
3. Determine $\overline{\Phi}_0$;
4. Determine $\Phi_1\ \overline{\Phi}_0$;
5. Determine $E_x = \Omega(\Phi_1\, \overline{\Phi_0})$ or $E_{\bar{x}} = \Omega(\Phi_0\, \overline{\Phi_1})$ depending on the type of event.

Step 5 must be applied as many times as the number of time instants in which the mission time is subdivided.

Finally the integration of $E_x = \Omega(\Phi_1\,\overline{\Phi_0})$ and $E_{\bar{x}} = \Omega(\Phi_0\,\overline{\Phi_1})$ gives the importance indexes. Since this operation is time consuming the following faster methods can be applied:

- If all initiating events are not repairable, then the importance measures for unavailability can be used also for unreliability since in this case $W_S(t) = Q_S(t)$;
- If the failure frequencies of the numerator and denominator of equations 5.8 and 5.10 are almost constants then the integration can be avoided in that $W(t) = \Omega(t)\ T$, which means that $IE_x = q_x\ \Omega_x / \Omega_S$ and $IE_{\bar{x}} = (1 - q_x)\ \Omega_{\bar{x}} / \Omega_S$.

The implementation of the above procedure in ASTRA 3.0 is straightforward, but it may be time consuming on large fault trees if the integration of the frequency functions must be performed.

To reduce the computation time the following approximated method may be considered.

Approximated method to determine the importance of enabling events

The LBDD of a function can be represented as follows:

$$\Phi(\mathbf{x}) = \bigvee_{k=1}^{N_x} R_{xk\to T}\left[\ x_{xk}\, D_{1xk} + \overline{x_{xk}}\ D_{0xk}\ \right] + S \qquad (5.15)$$

where: N_x is the number of occurrences of nodes with x; D_{1xk} and D_{0xk} are Boolean functions, i.e. the LBDD descending from the k-th occurrence (node) of x; $R_{xk\to T}$ is the disjunction of all paths from the k-th occurrence of x to the root of the LBDD; S is the disjunction of all paths not containing x.

Hence:

$$\Phi_1(\mathbf{x}) = \bigvee_{k=1}^{N_x} R_{xk\to T}\ D_{1xk} + S_k$$

$$\Phi_0(\mathbf{x}) = \bigvee_{k=1}^{N_x} R_{xk\to T}\ D_{0xk} + S_k$$

It is easy to see that: $\Phi_1\ \overline{\Phi_0} = \bigvee_{k=1}^{N_x} R_{xk\to T}\ D_{1xk}\ \overline{D_{0xk}}\ \overline{S}_k$.

R_k is independent from D_{1k}, D_{0k}, whereas S_k may share common events with R_k, D_{1k} and D_{0k}. A first simplifying hypothesis is that S_k is independent from R_k.

Therefore:

$$E_x = \sum_{k=1}^{Nx} \Omega(R_{xk\to T}\ D_{1xk}\ \overline{D_{0xk}}\ \overline{S_k}) \approx \sum_{k=1}^{Nx} [\Omega(R_{xk\to T})\ P(D_{1xk}\ \overline{D_{0xk}}\ \overline{S}_k) + P(R_{xk\to T})\ \Omega(D_{1xk}\ \overline{D_{0xk}}\ \overline{S}_k)]$$

In order to further simplify the calculation the hypothesis of independence of S from D_1 and D_0 cal also be assumed. This means that E_x is (non-conservatively) approximated by:

$$E_x \geq \sum_{k=1}^{Nx} [\Omega(R_{kx\to T})\ P(D_{1xk}\ \overline{D_{0xk}}) + P(R_{kx\to T})\ \Omega(D_{1xk}\ \overline{D_{0xk}})]\, P(\overline{S}_k) \qquad (5.16)$$

It can be shown that $S_k = \Phi \setminus R_{xk\to T}\ (x\, D_{1xk} + \bar{x}\, D_{0xk})$.

Passing to probabilities:

$$P(S_k) = P(\Phi) - P(R_{xk\to T})\,[q_x\ P(D_{1xk}) + (1 - q_x)\ P(D_{0xk})] \tag{5.17}$$

An alternative hypothesis is to set $P(\overline{S_k}) = 1$, which means that $P(S_k) = 0$ and $\Omega(\overline{S_k}) = \Omega(S_k) = 0$. Under this hypothesis the approximated E_x value is given by:

$$E_x \approx \Omega(R_{xk\to T})\ P(D_{1xk}\ \overline{D_{0xk}}) + P(R_{xk\to T})\ \Omega(D_{1xk}\ \overline{D_{0xk}}) \tag{5.18}$$

For each basic event the determination of the importance measures requires as many BDD traversing as the number of time points.

Determination of $\Omega(D_1\ \overline{D_0})$ *and* $P(D_1\ \overline{D_0})$

In practice, for each occurrence of x the BDD of $D_1\ \overline{D_0}$ is determined from which $P(D_1\ \overline{D_0})$ and $\Omega(D_1\ \overline{D_0})$ are obtained by means of the application of the following equations, applied visiting the LBDD in a bottom-up way. Note that the frequency of negated variables is set to zero.

If y is of SP type:

$Y = y\ F + G$

$Q_{out} = q_y\ Q_1 + (1 - q_y)\ Q_0$

If y is initiator then

$\omega_{out} = \omega_y\ (Q_1 - Q_0) + q_y\ \omega_1 + (1 - q_y)\ \omega_0$ (5.19)

else

$\omega_{out} = q_y\ \omega_1 + (1 - q_y)\ \omega_0$

If y is of SN type:

$Y = \$y\ F + G$

$Q_{out} = q_{\$y}\ Q_1 + (1 - q_{\$y})\ Q_0$ (5.20)

$\omega_{out} = q_{\$y}\ \omega_1 + (1 - q_{\$y})\ \omega_0$ for both initiator and enabler

If y is of DF type:

$Y = y\ F + \overline{y}\ G$

$Q_{out} = q_y\ Q_1 + (1 - q_y)\ Q_0$ (5.21)

If y is initiator then

$\omega_{out} = \omega_y\ Q_1 + q_y\ \omega_1 + (1 - q_y)\ \omega_0 - \omega_y\ \Pr\{F \wedge G\}$

else

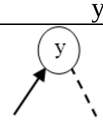
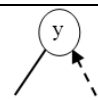
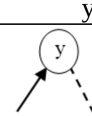
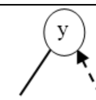
$\omega_{out} = q_y\ \omega_1 + (1 - q_y)\ \omega_0$ for both initiator and enabler

The above equations (5.19-21) can also be applied to the BDD of Φ for determining $\Omega(\Phi_1\ \overline{\Phi_0})$, $\Omega(\Phi_0\ \overline{\Phi_1})$

Quantification of $\Omega(R_{xk\to T})$ and $P(R_{xk\to T})$

For the quantification of these parameters it is necessary to consider the type of variable (y) as described in the Table below. In this case it is also necessary to consider whether the variable under consideration is initiating or enabling. In the first case the value of the failure frequency is different from zero for SP and DF events only if the event is initiating.

Table 5.3. Values assumed by the variables for calculating P(R) and Ω(R).

Var. type	y is Initiating event		y is Enabling event	
	 F G Left branch	 F G Right branch	 F G Left branch	 F G Right branch
SP	$R = y$	$R = \bar{y}$	$R = y$	$R = \bar{y}$
	$P(R) = q_y\, \Omega(F)$	$P(R) = (1 - q_y)\, \Omega(G)$	$P(R) = q_y\, \Omega(F)$	$P(R) = (1 - q_y)\, \Omega(G)$
	$\Omega(R) = \omega_y\, P(F)$	$\Omega(R) = -\, \omega_y\, P(G)$	$\Omega(R) = 0$	$\Omega(R) = 0$
SN	$R = \$y$	$R = \$y$	$R = \$y$	$R = \$y$
	$P(R) = (1 - q_y)\, \Omega(F)$	$P(R) = q_y\, \Omega(G)$	$P(R) = 1 - q_y$	$P(R) = q_y\, \Omega(G)$
	$\Omega(R) = \Omega(F)$	$\Omega(R) = \Omega(G)$	$\Omega(R) = 0$	$\Omega(R) = 0$
DF	$R = y$	$R = \bar{y}$	$R = y$	$R = \bar{y}$
	$P(R) = q_y\, \Omega(F)$	$P(R) = (1 - q_y)\, \Omega(G)$	$P(R) = q_y$	$P(R) = (1 - q_y)\, \Omega(G)$
	$\Omega(R) = \omega_y\, [P(F) - P(F \wedge G)]$	$\Omega(R) = \Omega(G)$	$\Omega(R) = 0$	$\Omega(R) = 0$

5.6.3 Determination of RAW and RRW

To determine RAW and RRW it is necessary to determine $\Omega(\Phi_{1x})$ and $\Omega(\Phi_{0x})$

Determination of $\Omega(\Phi_{0x})$

Let x be the variable for which the RAW and RRW are to be determined and y the current variable. The dependence on time is omitted in order to use a simpler notation.

If $y \neq x$ then:

The variable y represents an initiating event.

If y is of SP type:

$$Y = y\, F + G$$

$$Q_{out} = q_y\, Q_1 + (1 - q_y)\, Q_0$$

$$\omega_{out} = \omega_y\, Q_1 + q_y\, \omega_1 + (1 - q_y)\, \omega_0 - \omega_y\, Q_0$$

If y is of SN type:

$$Y = \$y\, F + G$$

$$Q_{out} = q_y\, Q_1 + (1 - q_y)\, Q_0$$

$$\omega_{out} = \nu_y\, Q_1 + (1 - q_y)\, \omega_1 + q_y\, \omega_0 - \nu_y\, Q_0$$

If y is of DF type:

$$Y = y\, F + \bar{y}\, G$$

$$Q_{out} = q_y\, Q_1 + (1 - q_y)\, Q_0$$

$$\omega_{out} = \omega_y\, Q_1 + q_y\, \omega_1 + (1 - q_y)\, \omega_0 + \nu_y\, Q_0 - (\omega_y + \nu_y)\, \Pr\{F \wedge G\}$$

The variable y represents an enabling event
An event is enabler if: 1) it is flagged as protective; and 2) it has $\lambda = 0$.
In this case the failure frequency is zero. The unavailability equation is the same as before.
Concerning the failure frequency:

If y is of SP type: $\omega_{out} = q_y\, \omega_1 + (1 - q_y)\, \omega_0$

If y is of SN type: $\omega_{out} = (1 - q_y)\, \omega_1 + q_y\, \omega_0$

If y is of DF type: $\omega_{out} = q_y\, \omega_1 + (1 - q_y)\, \omega_0$

If y = x then x = 1; $q_x=1$; $\omega_x = \nu_x = 0$; $q_{\$x}=0$; $\omega_{\$x} = \nu_{\$x} = 0$.

If y is of SP type:

$Q_{out} = Q_1$

$\omega_{out} = \omega_1$

If y is of SN type:

$Q_{out} = Q_0$

$\omega_{out} = \omega_0$

If y is of DF type:

$Q_{out} = Q_1$

$\omega_{out} = \omega_1$

At the root node $Q_{out} = P(\Phi_{1x})$ and $\omega_{out} = \Omega(\Phi_{1x})$.

Determination of $\Omega(\Phi_{0x})$
Let x be the variable for which the RAW and RRW are to be determined and y the current variable. The dependence on time is omitted in order to use a simpler notation.

If $y \neq x$ the equations to be applied are those above described.
If y = x then x = 0; $q_x=0$; $\omega_x = \nu_x = 0$; $q_{\$x}=1$; $\omega_{\$x} = \nu_{\$x} = 0$.

If y is of SP type:

$Q_{out} = Q_0$

$\omega_{out} = \omega_0$

If y is of SN type:

$Q_{out} = Q_1$

$\omega_{out} = \omega_1$

If y is of DF type:

$Q_{out} = Q_0$

$\omega_{out} = \omega_0$

At the root node $Q_{out} = P(\Phi_{0x})$ and $\omega_{out} = \Omega(\Phi_{0x})$.

5.7 Application of the Importance Measures to a Non-coherent fault tree

To show the result of the developed importance measures for initiating and enabling events a simple system taken from Beeson-Andrew (2003b) is considered. The schematic diagram of this system is presented in Figure 5.3.

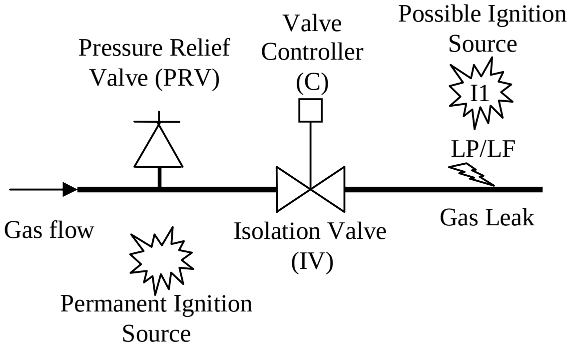


Figure 5.3. Leak-protection system

A leak in the high-pressure gas supply system beyond the isolation valve (IV) can occur due to the pipe leak (LP) or failure of the flange sealing (LF). In order to simplify the analysis the gas detection system is assumed perfectly reliable. In case of gas leak the isolation valve controller sends the signal to the isolation valve and closes it. In order to avoid the hammer effect on the isolation valve due to the high pressure - possibly resulting in pipe rupture before a valve - a pressure relief valve (PRV) is installed diverting the gas into a safe location outside permanent ignition source present close to the isolation valve.

The fault tree representing the system's failure is shown in Figure 5.4 in which, according to the ASTRA graphical notation, INH (Inhibit) gates are used to represent the combinations of initiating and enabler events, the latter acting on demand.
This fault tree has the following 8 prime implicants:

{LP IV I1}
{LP C I1}
{LF IV I1}
{LF C I1}
{LP PRV I1}
{LF PRV I1}
{LP $\overline{\text{IV}}$ $\overline{\text{C}}$ PRV}
{LF $\overline{\text{IV}}$ $\overline{\text{C}}$ PRV}

The parameters of basic events are provided in Table 5.4. Note that the possible ignition source, represented by the event I1, occurs once a week and lasts for 12 minutes. This has been modelled by BA as having a constant unconditional failure frequency of 1/840 (h^{-1}) and unavailability equal to 1/840. In ASTRA this event has been characterised as repairable with failure rate equal to 1/840 and repair time of 1 hour in order to have a constant unconditional failure frequency; in this way both the unconditional failure frequency and unavailability are equal to 1/840 as considered in the referenced paper.

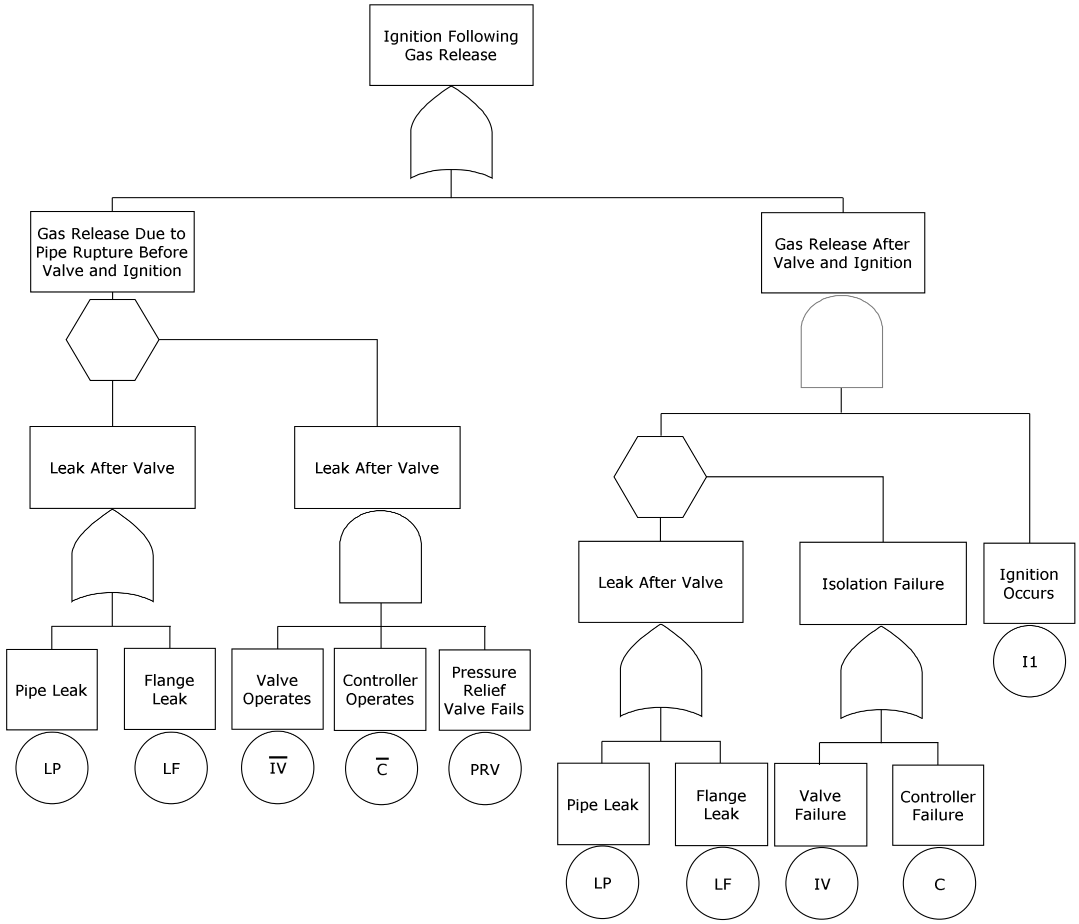


Figure 5.4. Fault tree of Leak-protection system's failure

Table 5.4. Failure rate, mean time to repair and inspection interval for the system components

Component	Failure rate, 1/h	MTTR, h	Inspection Interval, h
LF – leak from flange	1.80E-06	1	0
LP – leak from pipe	2.00E-08	1	0
IV – isolation valve	1.73E-05	20	8760
C – valve controller	5.00E-06	12	8760
I1 – ignition source	1/840	1	
PRV – relief valve	1.73E-06	20	8760

Before starting the analysis the role (initiating, enabling) that each component can have in a system-failure is analysed and taken into consideration. Based on the component's role the appropriate importance measures are calculated in order to assess component's contribution to the system's failure (top event).

According to the system's description three components (PRV, IV and C) are enablers, whereas LF, LP and I1 are initiators.

Table 5.5 contains the unavailability, unconditional failure and repair frequencies values for all basic events. Significant differences between these values and those presented in the BA paper concerns:

Unavailability of event C: in the paper the value 0.2196 indicates a printing mistake;

Unconditional repair frequencies of enabling events are wrongly determined in the paper on tested events.

Table 5.5. Component unavailability and unconditional failure and repair frequencies

Component	Unavailability	Unconditional failure frequency	Unconditional repair frequency
LF – leak from flange	1.799997E-06	1.799997E-06	1.799997E-06
LP – leak from pipe	2.000000E-08	2.000000E-08	2.000000E-08
IV – isolation valve	7.240800E-02	0.000000E+00	0.000000E+00
C - valve controller	2.164267E-02	0.000000E+00	0.000000E+00
I1 - ignition source	1.189060E-03	1.189060E-03	1.189060E-03
PRV – relief valve	7.240800E-02	0.000000E+00	0.000000E+00

In order to make calculation results comparable to the one provided in the Beeson-Andrews (2003) paper an attempt was made to use identical parameters. The mission time was not indicated in the article so 87,600 h was assumed for the mission time. Note that since all events' data represent repairable and tested components the unavailability value is not affected by the mission time; on the contrary the Expected Number of Failures $W_S(t)$ strongly depends on it.
The results obtained at system level are as follows:

- System unavailability Q_S = 1.198E-07;
- Expected Number of Failures W_S(87,600) = 1.051E-02.

The results at system level Q_S and W_S are not provided in the BA paper.

The results obtained on importance indexes from applying ASTRA are given in Table 5.6. The last column contains the total importance values: the ranking is given between brackets.
Concerning initiators the most significant contribution to the system's failure is given by LF, while LP is ranked 2nd and I1 is ranked as a least likely event to cause system's failure.
Among the enabling events the most importance one is related to the failure of the pressure relief valve, followed by the negated events. In our opinion the negated events should not be considered for design improvements because they represent conditions that must be satisfied for the occurrence of the Top event (in our case explosion). Therefore the top event frequency can be reduced by:

- reducing the unavailability of the pressure relief valve, e.g. by reducing the test interval; and/or
- by reducing the frequency of flange leaks.

A comparison of results from applying the ASTRA method and the BA method can be seen in Table 5.7. The main concern was raised by the difference of enabler importance measures for IV, C and $\overline{C}$.

In case of C and $\overline{\text{C}}$ the strong difference could be due to a mistake in the compilation of the table (the values could have been wrongly exchanged). The hand calculation performed resulted in the importance of C as equal to 6.38E-04. Taking into account the approximation introduced it can be stated that the correct result is that of ASTRA.
Moreover from Table 5.7 is can be verified that the sum of the importance measures for initiating events correctly sums to 1 for ASTRA; the sum of the values in the BA paper gives 0.97923, which is not correct.

In order to check the correctness of the importance measures for positive events provided in Beeson-Andrews (2003b) additional calculation was performed. From the original fault tree two negated events $\overline{IV}$ and $\overline{C}$ were removed making the fault tree coherent. The calculated importance values for both initiators and enablers for the coherent fault tree are shown in Table 5.8. This confirms the correct behaviour of ASTRA 3.0.

Table 5.6. ASTRA results for the various importance measures

Event	IB (eq. 5. 3 – 5.5)	II (eq. 5. 7)	IE (eq. 5.8)	Total II + IE
LF	6.582142E-02	9.873628E-01	2.729906E-03	9.900927E-01 (2)
LP	6.582130E-02	1.097070E-02	3.033229E-05	1.100103E-02 (4)
I1	1.683198E-07	1.667760E-03	1.667760E-03	3.333552E-03 (5)
IV	1.963944E-09		2.369955E-03	2.369955E-03 (6)
$\overline{IV}$	1.287769E-07		9.954808E-01	9.954808E-01 (3)
C	1.862038E-09		6.716204E-04	6.716204E-04 (7)
$\overline{C}$	1.220949E-07		9.954808E-01	9.954808E-01 (3)
PRV	1.651677E-06		9.966658E-01	9.966658E-01 (1)

Table 5.7. Comparison of the importance measures and their ranking

Event	Initiators		Enablers	
	ASTRA	[BA]	ASTRA	[BA]
LF	9.874E-01 (1)	9.571E-01 (1)	2.730E-03 (3)	5.790E-03 (5)
LP	1.097E-02 (2)	1.063E-02 (2)	3.033E-05 (6)	6.660E-05 (7)
I1	1.668E-03 (3)	5.830E-03 (3)	1.668E-03 (5)	5.700E-03 (5)
IV			2.370E-03 (4)	9.976E-01 (3)
$\overline{IV}$			9.955E-01 (2)	9.621E-01 (4)
C			6.716E-04 (5)	9.998E-01 (2)
$\overline{C}$			9.955E-01 (2)	4.350E-04 (6)
PRV			9.967E-01 (1)	9.999E-01 (1)

Table 5.8. Comparison of the importance measures for coherent system

Event	Initiators		Enablers	
	Coherent	Non-coherent	Coherent	Non-coherent
LF	9.876E-01 (1)	9.874E-01 (1)	1.389E-03 (4)	2.730E-03 (2)
LP	1.097E-02 (2)	1.097E-02 (2)	1.544E-05 (6)	3.033E-05 (6)
I1	1.405E-03 (3)	1.668E-03 (3)	1.405E-03 (3)	1.668E-03 (4)
IV			2.152E-03 (2)	2.370E-03 (3)
C			6.098E-04 (5)	6.716E-04 (5)
PRV			9.971E-01 (1)	9.967E-01 (1)

6. CONCLUSIONS

In this report we have described the methods implemented in ASTRA 3.0 to perform the importance analysis as part of the fault tree analysis procedure.
Equations for determining the importance measures for unavailability analysis for both coherent and non coherent fault trees have been described for the more general case of a modularised fault tree.

Among the importance measures that can be found in the scientific literature the following four have been considered for implementation in ASTRA 3.0:

- Probability of system critical state (equal to Birnbaum for coherent variables);
- Criticality
- Risk Achievement Worth
- Risk Reduction Worth

The well known Fussell-Vesely index has not been considered because its values for risk analysis applications are very close to the Criticality index values and also because it is related to the RRW, i.e. they present the same ranking.

These four indexes have been extended to the case of failure frequency analysis in which system components have different role and their failure can be categorised as initiating events and enabling events. The former events cause perturbation of process variables to critical values that require the intervention of the protective system. The failure of protective system components enable the perturbation to further propagate and eventually to lead to an accident.

The components importance ranking for unavailability and failure frequency for any importance measure are obviously different, except when all initiating events are non-repairable.

The literature on the importance measures for frequency analysis is not as rich as that of unavailability. Only few methods are available. A new method for the calculation of the importance indexes is described in this report. For initiating events the importance measure coincides with that of Barlow-Proschan, whether the importance measure for enabling events or for the enabling contribution of initiating events is based on a novel method. The comparison of our method with the exact method developed by Beeson-Andrews shows a complete agreement.

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REFERENCES

Barlow R. Proschan F. (1975), Importance of System Components and Fault Tree Events, Stochastic Processes and their Applications, vol. 3, pp 153-173.

Beeson S., Andrews J.D., (2003a), Birnbaum Measure of Component Importance for Non-coherent Systems, IEEE Transaction of Reliability, Vol.R52, N.2.

Beeson S., Andrews J.D., (2003b), Importance Measures for Non-Coherent-System Analysis, IEEE Transaction on Reliability, Vol. 52, N. 3, pp 301-310.

Becker, G., Camarinopoulos, L. (1993), Failure frequencies of non-coherent structures, *Reliability Engineering and System safety*, Vol. 41: 209-215.

Clarotti C. A., (1981), Limitation of Minimal Cut Set Approach in Evaluating Reliability of Systems with Repairable Components, IEEE Trans. Reliability, Vol. R-30, pp335-338.

Contini S., Cojazzi G.G.M., De Cola G., (2006), On the exact analysis of non coherent fault trees: the ASTRA package, PSAM 8, New Orleans, USA.

Contini, S., Cojazzi, G.G.M., Renda G., (2008), On the use of non-coherent fault trees in safety and security studies, *Reliability Engineering and System safety, V.93, N.12.*

Contini S., Matuzas V., (2009), ASTRA 3.0: Test Case Report, EUR 24124, ISBN 978-92-79-14608-4, ISSN 1018-5593, DOI 10.2788/51332.

Contini S., Matuzas V., (2010a), ASTRA 3.0: Logical and Probabilistic Analysis methods, EUR 24152, ISBN 978-92-79-14857-6, ISSN 1018-5593, DOI 10.2788/55214.

Contini S., Matuzas V., (2010b), Reduced ZBDD construction algorithms for large fault tree analysis, ESREL 2010, Greece (to be published).

Contini S., Fabbri L., Matuzas V., (2010c), Sensitivity Analysis Applied to Multiple Fault Tree, Chemical Engineering Transactions, 19, 225-230 DOI: 10.3303/CET1019037.

Demichela M., Piccinini N., Ciarambino I., Contini S., (2003), “On the numerical solution of fault trees” Reliability Eng., Sys, Safety, Vol 82, pp 141-147

Dutuit Y., Rauzy A., (2001), Efficient algorithms to assess component and gate importance in fault tree analysis, *Reliability Engineering and System Safety*, 72,

Kumamoto H., Henley E.J., (1996), Probabilistic Risk Assessment and Management for Engineers and Scientists, IEEE Press, New York.

IAEA (1991), Case study on the use of PSA methods: determining safety importance of systems and components at nuclear power plants, IAEA TECDOC 590, ISSN 1011-4289, Vienna

Jackson P.S., (1983), On the s-Importance of Elements and Prime Implicants of Non-Coherent Systems, , IEEE Transaction on Reliability, Vol.R32 No 1.

Lambert H. E., (1975), Measures of importance of events and cut sets in fault trees, Reliability and Fault Tree Analysis, SIAM Philadelphia.

Van der Borst M., Schoonakker, H. (2001), An overview of PSA importance measures, *Reliability Engineering and System Safety*, Vol . 72.

Zhang Q., Mei Q., (1985), Elements Importance and System failure Frequency of a 2-State Systems, IEEE Transaction on Reliability, 1985 Vol.R34 No 2.

APPENDIX 1

Determination of RAW for different types of variables

Let x be an *SP variable* and $\Phi(\mathbf{x}) = x\, \Phi(1, \mathbf{x}) + \Phi(0, \mathbf{x})$

$Q_S(t) = q_x(t))\, Q_S(t)|_{x=1} + (1 - q_x(t))\, Q_S(t)|_{x=0}$

$p^f{}_x(t) = Q_S(t)|_{x=1} – Q_S(t)|_{x=0}$ which is *valid only for SP events*

$Q_S(t)|_{x=0} = Q_S(t)|_{x=1} - p^f{}_x(t)$ which, substituted into

$Q_S(t) = q_x(t)\, Q_S(t)|_{x=1} + (1 - q_x(t))\, Q_S(t)|_{x=0}$ gives: $Q_S(t)|_{x=1} = Q_S(t) + (1- q_x(t))\, p^f{}_x(t)$

Therefore, $RAW_x(t) = Q_S(t)|_{x=1} / Q_S(t)$ becomes:

$RAW_x(t) = [Q_S(t) + (1- q_x(t))\, p^f{}_x(t)\,] / Q_S(t) =$

$\mathbf{RAW_x(t) = 1 + (1- q_x(t))\, p^f{}_x(t) / Q_S(t)}$

Let x be an *SN variable* and $\Phi(\mathbf{x}) = \$x\, \Phi(1, \mathbf{x}) + \Phi(0, \mathbf{x})$

$Q_S(t) = q_{\$x}(t))\, Q_S(t)|_{\$x=1} + (1 - q_{\$x}(t))\, Q_S(t)|_{\$x=0}$

$P^f{}_{\$x}(t) = Q_S(t)|_{\$x=1} – Q_S(t)|_{\$x=0}$ which is *valid only for SN events*

$Q_S(t)|_{\$x=0} = Q_S(t)|_{\$x=1} - p^f{}_{\$x}(t)$ which, substituted into

$Q_S(t) = q_{\$x}(t)\, Q_S(t)|_{\$x=1} + (1 - q_{\$x}(t))\, Q_S(t)|_{\$x=0}$ gives: $Q_S(t)|_{\$x=1} = Q_S(t) + (1- q_{\$x}(t))\, p^f{}_{\$x}(t)$

Therefore, $RAW_{\$x}(t) = Q_S(t)|_{\$x=1} / Q_S(t)$ becomes:

$RAW_{\$x}(t) = [Q_S(t) + (1- q_{\$x}(t))\, p^f{}_{\$x}(t)\,] / Q_S(t) =$

$RAW_{\$x}(t) = 1 + (1- q_{\$x}(t))\, p^f{}_{\$x}(t) / Q_S(t)$

Since $q_{\$x}(t) = 1 - q_x(t)$ and $p^f{}_{\$x}(t) = p^r{}_x(t)$ then RAW can be re-written as :

$\mathbf{RAW_{\$x}(t) = 1 + q_x(t)\, p^r{}_x(t) / Q_S(t)}$

Let x be a *DF variable*. In this case both the positive and negated contribution are calculated as above described.

Determination of RRW for different types of variables

Let x be a *SP variable* and $\Phi(\mathbf{x}) = x\, \Phi(1, \mathbf{x}) + \Phi(0, \mathbf{x})$

$Q_S(t) = q_x(t))\, Q_S(t)|_{x=1} + (1 - q_x(t))\, Q_S(t)|_{x=0}$

$p^f_x(t) = Q_S(t)|_{x=1} - Q_S(t)|_{x=0}$

$Q_S(t)|_{x=1} = Q_S(t)|_{x=0} + p^f_x(t)$ which, substituted into

$Q_S(t) = q_x(t)\ Q_S(t)|_{x=1} + (1 - q_x(t))\ Q_S(t)|_{x=0}$ gives: $Q_S(t)|_{x=0} = Q_S(t) - q_x(t)\ p^f_x(t)$

Therefore, $RRW_x(t) = Q_S(t) / Q_S(t)|_{x=0}$ becomes:

$RRW_x(t) = [Q_S(t) / [Q_S(t) - q_x(t)\ p^f_x(t)] =$

$\mathbf{RRW_x(t) = 1 / [1 - q_x(t)\ p^f_x(t) / Q_S(t)]}$

Let x be a *SN variable* and $\Phi(\mathbf{x}) = \$x\ \Phi(1, \mathbf{x}) + \Phi(0, \mathbf{x})$

$Q_S(t) = q_{\$x}(t))\ Q_S(t)|_{\$x=1} + (1 - q_{\$x}(t))\ Q_S(t)|_{\$x=0}$

$p^f_{\$x}(t) = Q_S(t)|_{\$x=1} - Q_S(t)|_{\$x=0}$

$Q_S(t)|_{\$x=1} = Q_S(t)|_{\$x=0} + p^f_{\$x}(t)$ which, substituted into

$Q_S(t) = q_{\$x}(t)\ Q_S(t)|_{\$x=1} + (1 - q_{\$x}(t))\ Q_S(t)|_{\$x=0}$ gives: $Q_S(t)|_{\$x=0} = Q_S(t) - q_{\$x}(t)\ p^f_{\$x}(t)$

Therefore, $RRW_{\$x}(t) = Q_S(t) / Q_S(t)|_{\$x=0}$ becomes:

$RRW_{\$x}(t) = Q_S(t) / [Q_S(t) - q_{\$x}(t)\ p^f_{\$x}(t)] =$

$RRW_{\$x}(t) = 1 / [1- q_{\$x}(t)\ p^f_{\$x}(t) / Q_S(t)]$

Since $q_{\$x}(t) = 1 - q_x(t)$, and $p^f_{\$x}(t) = p^r_x(t)$ then

$\mathbf{RRW_{\$x}(t) = 1 / [1 - (1 - q_x(t))\ p^r_x(t) / Q_S(t)]}$

Le x be a *DF variable*. In this case both the positive and negated contributions are calculated as above described.
Finally, it can be seen that:

$$RAW_{\&x} = RAW_x \qquad RRW_{\&x} = RRW_x \qquad RAW_{\overline{\&x}} = RAW_{\$x} \qquad RRW_{\overline{\&x}} = RRW_{\$x}$$

APPENDIX 2

Determination of p_x^f and p_x^r on a modularised fault tree

Let $\Phi = M\,\Phi_1 + \overline{M}\,\Phi_0 + \Phi_1\,\Phi_0$

be the non-coherent function Φ expanded with respect to the module M where Φ_1 and Φ_0 are the residues.

Let $M = x\,M_1 + \overline{x}\,M_0 + M_1\,M_0$

be the function of the module $M(\mathbf{x})$ expanded with respect to the variable x and

$\overline{M} = x\,\overline{M_1} + \overline{x}\,\overline{M_0} + \overline{M_1}\,\overline{M_0}$ its complemented form.

Passing to probabilities:

$$P(\Phi) = P(M)\,[P(\Phi_1) - P(\Phi_1\,\Phi_0) + P(\overline{M})\,[P(\Phi_0) - P(\Phi_1\,\Phi_0)] + P(\Phi_1\,\Phi_0)$$

$$P(M) = P(x)\,[P(M_1) - P(M_1\,M_0)] + P(\overline{x})\,[P(M_0) - P(M_1\,M_0)] + P(M_1\,M_0)$$

$$P(\overline{M}) = P(x)\,[P(\overline{M_1}) - P(\overline{M_1}\,\overline{M_0})] + P(\overline{x})\,[P(\overline{M_0}) - P(\overline{M_1}\,\overline{M_0})] + P(\overline{M_1}\,\overline{M_0})$$

The above equations can also be written as:

$$P(\Phi) = P(M)\,P(\Phi_1\,\overline{\Phi_0}) + P(\overline{M})\,P(\Phi_0\,\overline{\Phi_1}) + P(\Phi_1\,\Phi_0)$$

$$P(M) = P(x)\,P(M_1\,\overline{M_0}) + P(\overline{x})\,P(M_0\,\overline{M_1}) + P(M_1\,M_0)$$

$$P(\overline{M}) = P(x)\,P(\overline{M_0}\,M_1) + P(\overline{x})\,P(\overline{M_0}\,M_1) + P(\overline{M_1}\,\overline{M_0})$$

Now, the importance of $x \in \Phi$ is obtained when:

$(x \in M$ and $M \in \Phi)$ or $(\overline{x} \in M$ and $\overline{M} \in \Phi)$

Analogously, the importance of $\overline{x} \in \Phi$ is obtained when:

$(x \in M$ and $\overline{M} \in \Phi)$ or $(\overline{x} \in M$ and $M \in \Phi)$

Indicating with p_x^f the probability of the system critical state for the failure of x with respect to Φ, and with p_x^r the probability of the system critical state for the repair of x with respect to Φ, we can write:.

1) For the importance of $x \in \Phi$:

$$p_x^f = \frac{\partial P(\Phi)}{\partial P(M)}\frac{\partial P(M)}{\partial(x)} + \frac{\partial P(\Phi)}{\partial P(\overline{M})}\frac{\partial P(M)}{\partial P(\overline{x})} =$$

$$= P(\Phi_1\,\overline{\Phi_0})\,P(M_1\,\overline{M_0}) + P(\Phi_0\,\overline{\Phi_1})\,P(M_0\,\overline{M_1}) = p_M^f\;p_x^{fM} + p_{\overline{M}}^f\;p_{\overline{x}}^{fM}$$

2) For the importance of $\overline{x} \in \Phi$:

$$p_{\overline{x}}^f = \frac{\partial P(\Phi)}{\partial P(\overline{M})}\frac{\partial P(M)}{\partial P(x)} + \frac{\partial P(\Phi)}{\partial P(M)}\frac{\partial P(M)}{\partial P(\overline{x})} =$$

$$= P(\Phi_0\,\overline{\Phi_1})\,P(M_1\,\overline{M_0}) + P(\Phi_1\,\overline{\Phi_0})\,P(M_0\,\overline{M_1}) = p_{\overline{M}}^f\;p_x^{fM} + p_M^f\;p_{\overline{x}}^{fM}$$

Considering that $p_{\overline{x}}^f = p_x^r$ and vice versa, the above equation can also be written as:

$$p_x^f = p_M^f\;p_x^{fM} + p_M^r\;p_x^{rM} \qquad \text{(A2.1)}$$

$$p_x^r = p_M^r\ p_x^{fM} + p_M^f\ p_x^{rM} \tag{A2.2}$$

where

$p_M^f = P(\Phi_1\ \overline{\Phi_0})$ is the probability of the system critical state for the failure of M in Φ;

$p_M^r = P(\overline{\Phi_1}\ \ \Phi_0)$ is the probability of the system critical state for the repair of $\overline{M}$ in Φ.

$p_x^{fM} = P(M_1\ \overline{M_0})$ is the probability of the critical state of x in M.

$p_x^{rM} = P(\overline{M_1}\ M_0)$ is the probability of the critical state for the repair of x in M.

The dependence of time of the above equations is not represented for the sake of simplicity, but it is understood that p_x^f and p_x^r are calculated at a give time t.

Equations A2.1 and A2.2 are valid for DF variables; simpler relationships can be derived for SP and SN variables.

SP variables

The importance of $x \in \Phi$ is obtained when: $x \in M$ and $M \in \Phi$

Equations are derived from A2.1 and A2.2 considering that for coherent positive variables $\Phi_1\ \Phi_0 = \Phi_0$ and $M_1\ M_0 = M_0$; consequently $\overline{\Phi_1}\ \Phi_0 = 0$ and $\overline{M_1}\ M_0 = 0$. Hence

$$p_x^f = \frac{\partial P(\Phi)}{\partial P(\mathrm{M})}\frac{\partial P(M)}{\partial \mathrm{P(x)}} = P(\Phi_1\ \overline{\Phi_0})\ P(M_1\ \overline{M_0}) = p_M^f\ \ p_x^{fM} \tag{A2.3}$$

$$p_{\bar{x}}^f = 0 \tag{A2.4}$$

SN variables

Analogously, the importance of $\overline{x} \in \Phi$ is obtained when: $\overline{x} \in M$ and $M \in \Phi$

Equations are derived from A2.1 and A2.2 considering that for negated variables $\Phi_1\ \Phi_0 = \Phi_1$ and $M_1\ M_0 = M_1$; consequently $\overline{\Phi_0}\ \Phi_1 = 0$ and $\overline{M_0}\ M_1 = 0$. Hence

$$p_x^f = 0$$

$$p_x^r = \frac{\partial P(\Phi)}{\partial P(\mathrm{M})}\frac{\partial P(M)}{\partial \mathrm{P}(\overline{\mathrm{x}})} = P(\Phi_1\ \overline{\Phi_0})\ P(M_0\ \overline{M_1}) = p_M^f\ \ p_x^{rM}$$

APPENDIX 3

Determination of $\mathrm{I_x}$, $\mathrm{E_x}$, $\mathrm{I_{\overline{x}}}$, $\mathrm{E_{\overline{x}}}$ on a modularised fault tree

Given the more general case of a non-coherent function $\Phi = \mathrm{M}\,\Phi_1 + \overline{\mathrm{M}}\,\Phi_0 + \Phi_1\,\Phi_0$, where M is a module containing the DF variable x, and the function $\mathrm{M} = \mathrm{x}\,\mathrm{M}_1 + \overline{\mathrm{x}}\,\mathrm{M}_0 + \mathrm{M}_1\,\mathrm{M}_0$, the objectives is to find the relationships linking the importance of x in M and of M in Φ to find the importance of x in Φ in case of frequency analysis.

Since the variable is non-coherent we expect to obtain the positive and the negative contributions. The failure frequency of Φ is given by:

$$\begin{aligned}\Omega(\Phi) &= \Omega(\mathrm{M})\,\mathrm{P}(\Phi_1) + \mathrm{P}(\mathrm{M})\,\Omega(\Phi_1) + \Omega(\overline{\mathrm{M}})\,\mathrm{P}(\Phi_0) + \mathrm{P}(\overline{\mathrm{M}})\,\Omega(\Phi_0) + \Omega(\Phi_1\,\Phi_0) + \\ &\quad - \Omega(\mathrm{M})\,\mathrm{P}(\Phi_1\,\Phi_0) - \mathrm{P}(\mathrm{M})\,\Omega(\Phi_1\,\Phi_0) - \Omega(\overline{\mathrm{M}})\,\mathrm{P}(\Phi_1\,\Phi_0) - \mathrm{P}(\overline{\mathrm{M}})\,\Omega(\Phi_1\,\Phi_0)\end{aligned}$$

or equivalently as:

$$\Omega(\Phi) = \Omega(\mathrm{M})\,\mathrm{P}(\Phi_1\,\overline{\Phi_0}) + \mathrm{P}(\mathrm{M})\,\Omega(\Phi_1\,\overline{\Phi_0}) + \Omega(\overline{\mathrm{M}})\,\mathrm{P}(\overline{\Phi_1}\,\Phi_0) + \mathrm{P}(\overline{\mathrm{M}})\,\Omega(\overline{\Phi_1}\,\Phi_0) + \Omega(\Phi_1\,\Phi_0)$$

For the module M in positive form we have (see Appendix 2):

$$\mathrm{M} = \mathrm{x}\,\mathrm{M}_1 + \overline{\mathrm{x}}\,\mathrm{M}_0 + \mathrm{M}_1\,\mathrm{M}_0$$

$$\mathrm{P}(\mathrm{M}) = \mathrm{P}(\mathrm{x})\,\mathrm{P}(\mathrm{M}_1\,\overline{\mathrm{M}_0}) + \mathrm{P}(\overline{\mathrm{x}})\,\mathrm{P}(\mathrm{M}_0\,\overline{\mathrm{M}_1}) + \mathrm{P}(\mathrm{M}_1\,\mathrm{M}_0)$$

$$\Omega(\mathrm{M}) = \Omega(\mathrm{x})\,\mathrm{P}(\mathrm{M}_1\,\overline{\mathrm{M}_0}) + \Omega(\overline{\mathrm{x}})\,\mathrm{P}(\mathrm{M}_0\,\overline{\mathrm{M}_1}) + \mathrm{P}(\mathrm{x})\,\Omega(\mathrm{M}_1\overline{\mathrm{M}_0}) + \mathrm{P}(\overline{\mathrm{x}})\,\Omega(\mathrm{M}_0\overline{\mathrm{M}_1}) + \Omega(\mathrm{M}_1\,\mathrm{M}_0)$$

For the module in negated form we have:

$$\overline{\mathrm{M}} = \overline{\mathrm{x}}\,\overline{\mathrm{M}_0} + \mathrm{x}\,\overline{\mathrm{M}_1} + \overline{\mathrm{M}_1}\,\overline{\mathrm{M}_0}$$

$$\mathrm{P}(\overline{\mathrm{M}}) = \mathrm{P}(\mathrm{x})\,\mathrm{P}(\overline{\mathrm{M}_0}\,\mathrm{M}_1) + \mathrm{P}(\overline{\mathrm{x}})\,\mathrm{P}(\overline{\mathrm{M}_0}\,\mathrm{M}_1) + \mathrm{P}(\overline{\mathrm{M}_1}\,\overline{\mathrm{M}_0})$$

$$\Omega(\overline{\mathrm{M}}) = \Omega(\overline{\mathrm{x}})\,\mathrm{P}(\mathrm{M}_1\,\overline{\mathrm{M}_0}) + \Omega(\mathrm{x})\,\mathrm{P}(\mathrm{M}_0\,\overline{\mathrm{M}_1}) + \mathrm{P}(\mathrm{x})\,\Omega(\mathrm{M}_0\overline{\mathrm{M}_1}) + \mathrm{P}(\overline{\mathrm{x}})\,\Omega(\mathrm{M}_1\overline{\mathrm{M}_0}) + \Omega(\overline{\mathrm{M}_1}\,\overline{\mathrm{M}_0})$$

Considering that the importance of $\mathrm{x} \in \Phi$ is obtained when:
($\mathrm{x} \in \mathrm{M}$ and $\mathrm{M} \in \Phi$) or ($\overline{\mathrm{x}} \in \mathrm{M}$ and $\overline{\mathrm{M}} \in \Phi$) we have respectively for initiating and enabling events that:

$$\begin{aligned}I_x &= \frac{\partial\Omega(\Phi)}{\partial\Omega(\mathrm{x})} = \\ &= \frac{\partial\Omega(\Phi)}{\partial\Omega(\mathrm{M})}\frac{\partial\Omega(\mathrm{M})}{\partial\Omega(\mathrm{x})} + \frac{\partial\Omega(\Phi)}{\partial\Omega(\overline{\mathrm{M}})}\frac{\partial\Omega(\mathrm{M})}{\partial\Omega(\overline{\mathrm{x}})} = \\ &= \mathrm{P}(\Phi_1\,\overline{\Phi_0})\,\mathrm{P}(\mathrm{M}_1\,\overline{\mathrm{M}_0}) + \mathrm{P}(\Phi_0\,\overline{\Phi_1})\,\mathrm{P}(\mathrm{M}_0\,\overline{\mathrm{M}_1}) = \mathrm{I_M}\,\mathrm{I_x^M} + \mathrm{I_{\overline{M}}}\,\mathrm{I_{\overline{x}}^M}\end{aligned} \qquad \text{(A3.1)}$$

$$\begin{aligned}E_x &= \frac{\partial\Omega(\Phi)}{\partial P(\mathrm{x})} = \\ &= \frac{\partial\Omega(\Phi)}{\partial\mathrm{P}(\mathrm{M})}\frac{\partial P(\mathrm{M})}{\partial P(\mathrm{x})} + \frac{\partial\Omega(\Phi)}{\partial P(\overline{\mathrm{M}})}\frac{\partial P(\mathrm{M})}{\partial P(\overline{\mathrm{x}})} = \\ &= \Omega(\Phi_1\,\overline{\Phi_0})\,\mathrm{P}(\mathrm{M}_1\,\overline{\mathrm{M}_0}) + \Omega(\Phi_0\,\overline{\Phi_1})\,\mathrm{P}(\mathrm{M}_0\,\overline{\mathrm{M}_1}) = \mathrm{E_M}\,\mathrm{I_x^M} + \mathrm{E_{\overline{M}}}\,\mathrm{I_{\overline{x}}^M}\end{aligned} \qquad \text{(A3.2)}$$

Analogously, the importance of $\overline{\mathrm{x}} \in \Phi$ is obtained when:
($\mathrm{x} \in \mathrm{M}$ and $\overline{\mathrm{M}} \in \Phi$) or ($\overline{\mathrm{x}} \in \mathrm{M}$ and $\mathrm{M} \in \Phi$)

$$I_{\bar{x}} = \frac{\partial\Omega(\Phi)}{\partial\Omega(\bar{\mathrm{x}})} =$$

$$= \frac{\partial\Omega(\Phi)}{\partial\Omega(\overline{\mathrm{M}})}\frac{\partial\Omega(\mathrm{M})}{\partial\Omega(\mathrm{x})} + \frac{\partial\Omega(\Phi)}{\partial\Omega(\mathrm{M})}\frac{\partial\Omega(\mathrm{M})}{\partial\Omega(\bar{\mathrm{x}})} =$$

$$= \mathrm{P}(\Phi_0\ \overline{\Phi_1})\ \mathrm{P}(\mathrm{M}_1\ \overline{\mathrm{M}_0}) + \mathrm{P}(\Phi_1\ \overline{\Phi_0})\ \mathrm{P}(\mathrm{M}_0\ \overline{\mathrm{M}_1}) = \mathrm{I}_{\overline{\mathrm{M}}}\ \mathrm{I}_{\mathrm{x}}^{\mathrm{M}} + \mathrm{I}_{\mathrm{M}}\ \mathrm{I}_{\bar{\mathrm{x}}}^{\mathrm{M}} \qquad \text{(A3.3)}$$

$$E_{\bar{x}} = \frac{\partial\Omega(\Phi)}{\partial P(\bar{\mathrm{x}})} =$$

$$= \frac{\partial\Omega(\Phi)}{\partial P(\overline{\mathrm{M}})}\frac{\partial P(\mathrm{M})}{\partial P(\mathrm{x})} + \frac{\partial\Omega(\Phi)}{\partial \mathrm{P}(\mathrm{M})}\frac{\partial P(\mathrm{M})}{\partial P(\bar{\mathrm{x}})} =$$

$$= \Omega(\Phi_0\ \overline{\Phi_1})\ \mathrm{P}(\mathrm{M}_1\ \overline{\mathrm{M}_0}) + \Omega(\Phi_1\ \overline{\Phi_0})\ \mathrm{P}(\mathrm{M}_0\ \overline{\mathrm{M}_1}) = \mathrm{E}_{\overline{\mathrm{M}}}\ \mathrm{I}_{\mathrm{x}}^{\mathrm{M}} + \mathrm{E}_{\mathrm{M}}\ \mathrm{I}_{\bar{\mathrm{x}}}^{\mathrm{M}} \qquad \text{(A3.4)}$$

Equations A3.1 and A3.4 are valid for DF variables; simpler relationships can be derived for SP and SN variables. They are reported here below for the sake of completeness.

SP variables

The importance of $\mathrm{x} \in \Phi$ is obtained when: $\mathrm{x} \in \mathrm{M}$ and $\mathrm{M} \in \Phi$

Equations are derived from A3.1 and A3.2 considering that for coherent positive variables $\Phi_1\ \Phi_0 = \Phi_0$ and $\mathrm{M}_1\ \mathrm{M}_0 = \mathrm{M}_0$, i.e. $\overline{\Phi_1}\ \Phi_0 = 0$ and $\overline{\mathrm{M}_1}\ \mathrm{M}_0 = 0$. Therefore:

$$I_x = \frac{\partial\Omega(\Phi)}{\partial\Omega(\mathrm{x})} = \frac{\partial\Omega(\Phi)}{\partial\Omega(\mathrm{M})}\frac{\partial\Omega(\mathrm{M})}{\partial\Omega(\mathrm{x})} = \mathrm{P}(\Phi_1\ \overline{\Phi_0})\ \mathrm{P}(\mathrm{M}_1\ \overline{\mathrm{M}_0}) = \mathrm{I}_{\mathrm{M}}\ \mathrm{I}_{\mathrm{x}}^{\mathrm{M}} \qquad \text{(A3.5)}$$

$$E_x = \frac{\partial\Omega(\Phi)}{\partial P(\mathrm{x})} = \frac{\partial\Omega(\Phi)}{\partial \mathrm{P}(\mathrm{M})}\frac{\partial P(\mathrm{M})}{\partial P(\mathrm{x})} = \Omega(\Phi_1\ \overline{\Phi_0})\ \mathrm{P}(\mathrm{M}_1\ \overline{\mathrm{M}_0}) = \mathrm{E}_{\mathrm{M}}\ \mathrm{I}_{\mathrm{x}}^{\mathrm{M}} \qquad \text{(A3.6)}$$

SN variables

Analogously, the importance of $\bar{\mathrm{x}} \in \Phi$ is obtained when: $\bar{\mathrm{x}} \in \mathrm{M}$ and $\mathrm{M} \in \Phi$

$$I_{\bar{x}} = \frac{\partial\Omega(\Phi)}{\partial\Omega(\bar{\mathrm{x}})} = \frac{\partial\Omega(\Phi)}{\partial\Omega(\mathrm{M})}\frac{\partial\Omega(\mathrm{M})}{\partial\Omega(\bar{\mathrm{x}})} = \mathrm{P}(\Phi_1\ \overline{\Phi_0})\ \mathrm{P}(\mathrm{M}_0\ \overline{\mathrm{M}_1}) = \mathrm{I}_{\mathrm{M}}\ \mathrm{I}_{\bar{\mathrm{x}}}^{\mathrm{M}} \qquad \text{(A3.6)}$$

$$E_{\bar{x}} = \frac{\partial\Omega(\Phi)}{\partial P(\bar{\mathrm{x}})} = \frac{\partial\Omega(\Phi)}{\partial \mathrm{P}(\mathrm{M})}\frac{\partial P(\mathrm{M})}{\partial P(\bar{\mathrm{x}})} = \Omega(\Phi_1\ \overline{\Phi_0})\ \mathrm{P}(\mathrm{M}_0\ \overline{\mathrm{M}_1}) = \mathrm{E}_{\mathrm{M}}\ \mathrm{I}_{\bar{\mathrm{x}}}^{\mathrm{M}} \qquad \text{(A3.7)}$$

APPENDIX 4

Complementation of an LBDD

In ASTRA 3 the fault tree is represented as a Labelled BDD, i.e. an OBDD in which the variables associated to the nodes are dynamically labelled with their type (Contini et al, 2008). Indeed a non-coherent fault tree may contain three different types of basic events or variables, namely:

1. normal or positive, e.g. x;
2. negated, e.g. $\overline{y}$;
3. events that appear both in positive and negated forms, e.g. z, $\overline{z}$.

In ASTRA the following definitions are used. Variables of type 1 are referred to as *Single form Positive variables* (SP), variables of type 2 as *Single form Negated variables* (SN), whereas variables of the third type as *Double Form variables* (DF). For instance, the function $\Phi = \overline{a}\ b + \overline{a}\ c + b\ \overline{c}$ contains the SN variable *a*, the SP variable *b* and the DF variable *c*.

Each negated variables $\overline{x}$ is represented as a labelled normal variable \$x. For instance, the function $\Phi = \overline{a}\ b + \overline{a}\ c + b\ \overline{c}$ is written as Φ = \$a b +\$a c + b \$c.

A coherent function contains only coherent variables, i.e. variables in positive form (SP).

A non coherent function contains also variables in negated form (SN, DF).

During the LBDD construction variables of DF type may be generated as a combination of two variables of different type.
The label associated to a variable (note that the same variable in two different nodes may have different labels) is the information used to apply the appropriate logical and probabilistic algorithms.

In this report the importance measures requires the complementation of Boolean function represented as LBDD. It is well known that the complementation of a BDD is obtained by complementing only the terminal nodes. The LBDD nodes are associated with the variables type. Hence the complementation of an LBDD needs to change also the variables types.

Given a function stored in the form of an LBDD its complemented form is obtained visiting it in Top-down mode and applying the following rules to each node, non terminal and terminal (the symbol ¬ means NOT):

¬&x = &x
¬x = \$x   exchanging its descendants
¬\$x = x   "   "   "
¬0 =1
¬1 = 0

The double form nodes are not changed. The SP and SN are changed plus their left and right descendants are exchanged. The simple inversion (complementation) is done for terminal nodes 1, 0.

As a simple example of the application of the above rules consider the following non-coherent function: $\Phi = a\ b\ d + a\ \overline{b}\ c + \overline{c}\ d\ e + a\ d\ e + a\ c\ d$ whose LBDD is shown on the left of Figure A.1.
The complemented form is on the right.

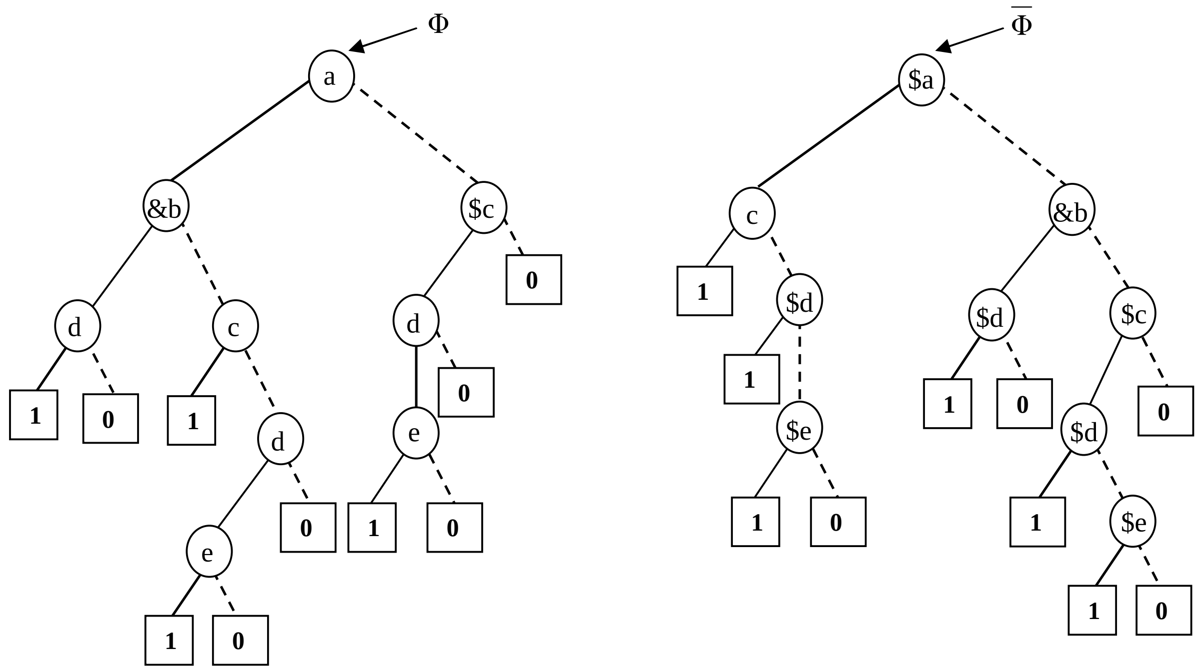

Figure A.1. Resulting function (on the left) obtained complementing the function on the right

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Abstract
This report deals with the problem of determining the exact values of the importance indexes of basic events in case of both unavailability and frequency analysis for coherent and non-coherent fault trees. In particular a new method is described for determining the importance of enabling events in case of frequency analysis. Insights are given into the importance analysis implemented in the new software ASTRA 3.0 based on the Binary Decision Diagram approach with Labelled variables (LBDD). The analysis methods are also described with reference to modularised fault trees. Simple numerical examples are provided to clarify how the methods work.

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